# AN INVESTIGATION OF LOW LAMBDA YIELD FAILURE DESIGN OF STIFFENED CYLINDRICAL PRESSURE VESSELS UNDER EXTERNAL HYDROSTATIC PRESSURE

Daniel George Hickey

AN INVESTIGATION OF LOW LAMBDA

YIELD FAILURE DESIGN OF STIFFENED

CYLINDRICAL PRESSURE VESSELS UNDER

EXTERNAL HYDROSTATIC PRESSURE

by

DANIEL GEORGE HICKEY

ENSIGN, UNITED STATES NAVY

B.S., United States Naval Academy (1971)

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE

IN NAVAL ARCHITECTURE AND MARINE ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June, 1972

AN INVESTIGATION OF LOW LAMBDA

YIELD FAILURE DESIGN OF STIFFENED

CYCLINDRICAL PRESSURE VESSELS UNDER

EXTERNAL HYDROSTATIC PRESSURE

by

Daniel G. Hickey

Submitted to the Department of Naval Architecture and Marine Engineering in partial fulfillment of the requirements for the degree of Master of Science in Naval Architecture and Marine Engineering.

## ABSTRACT

The designer of stiffened cylindrical pressure vessels is often faced with the problem of comparing structures of wiedly varying geometries. To do this, he makes use of a number of dimensionless parameters whose function is to reduce the number of variables in the design problem.

The object of this thesis was to investigate the effect of the dimensionless parameter, $\lambda$, on stiffened cylindrical pressure vessel design. In particular, the $\lambda$ region where collapse of vessels due to yield-initiated buckling between the frames was identified and delimited. This region was used as the first iteration in a design procedure aimed at producing a vessel of maximum structural efficiency. Design curves which may be used to develop such a vessel are provided for shell thicknesses up to $h/D_m = .01$. These curves are based on both maximum principle stress and Hencky-von Mises stress criteria and provision is made in a computer program for use of other methods. A modified Foppl Formula is used for frame evaluations.

The formulas used range from conservative to liberal in their approximations of collapse pressures. However, use of even the most conservative formulas results in structural efficiencies which compare favorably with present-day practices.

Thesis Supervisor: John Harvey Evans
Title: Professor of Naval Architecture

ACKNOWLEDGEMENTS

The author wishes to thank his thesis supervisor, Professor J. Harvey Evans, for suggesting this topic and providing direction when it was most needed.

Also, my deepest appreciation is extended to Jeanne Ellen Boyd, whose smiling countenance helped as much as her typing in the completion of this thesis.

## TABLE OF CONTENTS

LIST OF FIGURES

## NOMENCLATURE

$A$ Cross sectional area of frame, $in^2$

$A_b$ Cross sectional area of frame plus plating of width b, $in^2$

$A_1, A_2, A_3$ Plasticity coefficients, see Reference (10)

$b$ Width of frame at shell, in

$c$ Distance from neutral axis of bending to extreme outside fiber, in

$D$ Outside diameter of shell, in

$d$ Depth of rectangular frame, in

$D_b$ Diameter to neutral axis of frame plus plating of width b, in

$D_m$ Diameter to neutral axis of shell alone, in

$E$ Modulus of elasticity, $lbs/in^2$

$e$ Eccentricity, in

$E_s$ Secant modulus of elasticity, $lbs/in^2$

$f$ Width of flange of T frame, in

$h$ Shell thickness, in

$I_b$ Moment of inertia, frame plus plating of width b, $in^4$

$I_e$ Moment of inertia, frame plus plating of effective width, see Reference (10), $in^4$

$I_f$ Moment of inertia, frame alone, $in^4$

$I_1$ Moment of inertia, frame plus plating of width L, $in^4$

$L$ Unsupported span of plating, in

$L_b$ Bulkhead spacing, in

$L_f$ Frame spacing, in

$L_1$ Unsupported span of plating taken as frame sapcing, in

$n$ Number of complete waves in buckled configuration - circumferentially

$p_c$ Collapse pressure, $lbs/in^2$

$p_1$ Actual collapse pressure desired, $lbs/in^2$

$p_{c1}, p_{c2}, p_{c3}$ Temporary values of $p_c$ for iteration

$q_f$ Collapse load per unit axial length of a ring frame, $lbs/in^2$

$R$ Outside shell radius, in

$R_f$ Radius to neutral axis of frame, in

$R_m$ Radius to neutral axis of shell, in

$R_1$ Radius to neutral axis of frame plus shell of width L, in

$V$ Volume displaced by cylinder per unit length, $in^3/ft$

$Wt$ Weight per unit length, lbs/ft

$\Delta$ Weight of salt water displaced by cylinder per unit length, lbs/ft

$\eta$ Efficiency - defined in Section III

$\lambda$ Slenderness Ratio - defined in Section I

$\mu$ Poissons ratio

$\sigma_F$ Axial stress at mid-thickness, $lbs/in^2$

$\sigma_T$ Tangential (circumferential) stress at mid-thickness, $lbs/in^2$

$\sigma_y$ Material yield stress, $lbs/in^2$

$\psi$ Pressure factor - defined in Section I

# I. INTRODUCTION

Stiffened cylindrical pressure vessels under external hydrostatic pressure fail by one or any combination of three collapse modes. While both theoretical and empirical expressions have been developed to determine collapse pressures for all modes, it is up to the designer to determine what structural proportions will result in the most efficient use of material strength and stiffness properties for a required collapse pressure. Two dimensionless parameters which have been widely used to define different geometries and their capabilities are $\psi$ and $\lambda$. $\psi$ is called pressure factor and it is a ratio of a structure's actual collapse pressure to the collapse pressure of an unstiffened tube with the same $h/D_m$ ratio.

$$\psi = \frac{P_c}{2\frac{h}{D_m}\sigma_Y}$$

$\lambda$ is called slenderness ratio. It is analogous to the slenderness ratio, $l/r$, of column theory, however, it includes terms which allow for different material properties.

$$\lambda = \sqrt[4]{\frac{(L/D_m)^2}{(h/D_m)^3}} \sqrt{\frac{\sigma_Y}{E}}$$

Despite their fairly wide use by designers in comparisons of different structural geometries, this

researcher is unaware that any attempt has been made to develop a relationship between these parameters which would indicate their effect on structural efficiency. Also, it has been stated that these parameters are sometimes misleading (7), however, no definite limits on their usefulness have been established in the literature,and only a few other dimensionless parameters have been developed to be used instead. A need exists therefore, to define the limits of usefulness of both $\psi$ and $\lambda$ and to develop within those limits, relationships between them and structural efficiency which may be readily applied in the early design stages of a stiffened cylindrical shell.

In this report, two failure criteria, maximum circumferential stress and Hencky-von Mises, have been used to develop relationships between $\lambda$, $\psi$, and efficiency. In the course of the development, certain trends of the parameters have been noted and in the concluding remarks, useful limits of $\lambda$ and $\psi$ are indicated. In particular, the "low lambda" region has been defined and an introduction made to the concept of maximizing structural efficiency within that region. Design curves are presented to enable optimization within the low lambda region of structures with collapse pressures up to 2500 psi and examples of the process are given. Special note should be taken

of Appendices B and C where remarks concerning very low lambda design are made.

## II. DEVELOPMENT OF DESIGN CRITERIA

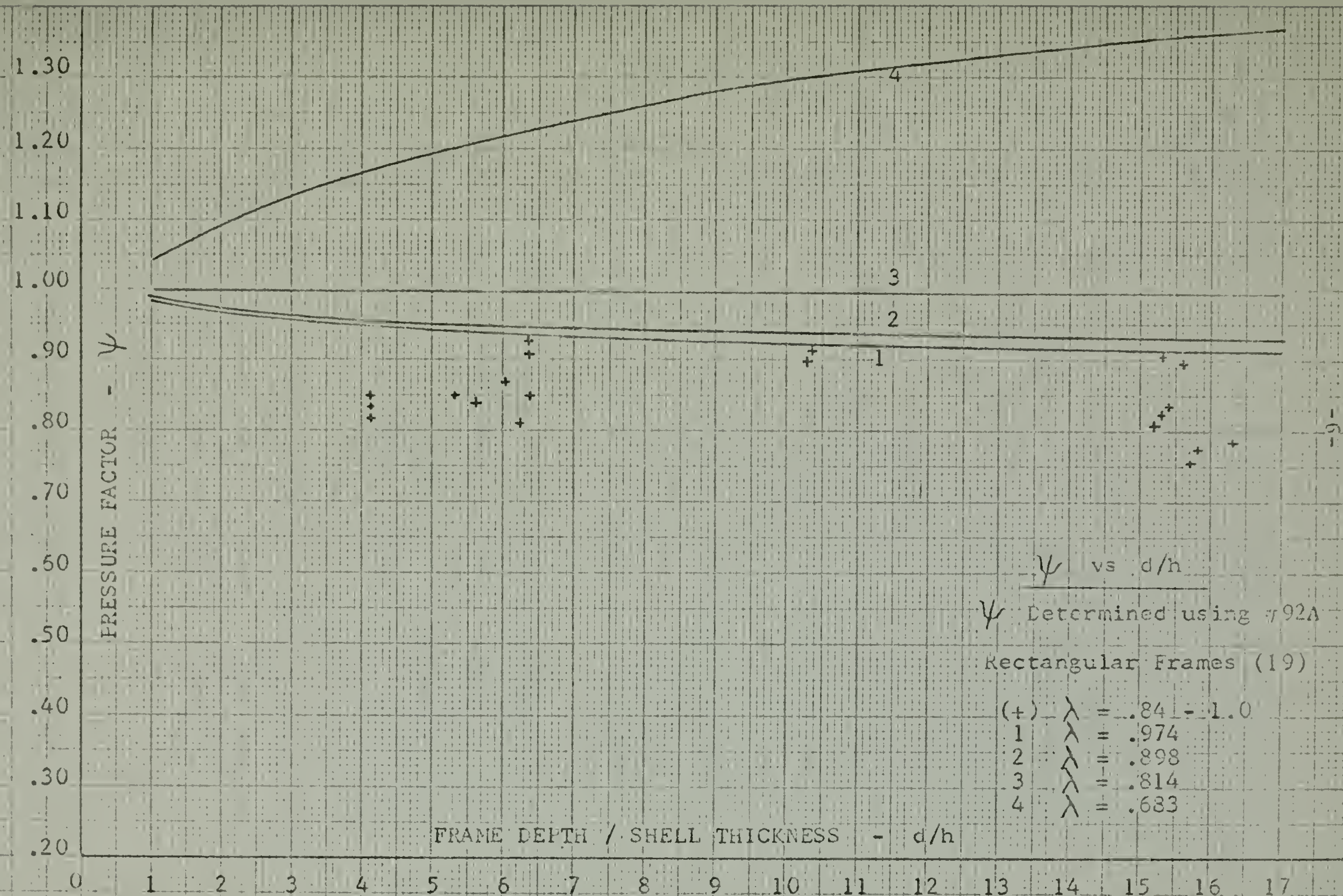
Historically, the largest and most critically designed stiffened cylindrical pressure vessels have been intended for use as submersibles. Such vessels, under external hydrostatic pressure, have been known to collapse in one, or a combination of three modes. Since the mechanism which precipitates failure in each mode is different, the designer must take all three modes into consideration when designing for a given collapse pressure. However, he has a choice in the design method he may use when developing a highly efficient structure. He may design toward one mode of collapse, seeking to eliminate the possibility of collapse in others while changing structural proportions to obtain a high efficiency. His alternative is to simply design toward the most efficient structure, not worrying about the mode of collapse as long as the required collapse pressure is obtained.

An example of the first method may be found in the design of the relatively shallow-diving submarines of World War II. These vessels were designed to collapse by asymmetric buckling between frames. It was felt that imminent collapse would be signalled by the appearance of lobes girdling the hull and the crew would be given a chance to surface before total collapse occurred (22).

This explanation indicates that for the volume-limited submarines of that era, efficiency criteria could be established to a large degree by safety considerations.

For deeper diving, weight-limited submarines, the first method proved inadequate and the second method was refined in a number of ways to allow development of highly efficient vessels. A refinement which will now be discussed is one which assumed that $\psi$ is limited to 1.0. This assumption used the maximum circumferential stress criterion and minimized weight by using the smallest possible number of frames. It resulted in a structure proportioned such that $\lambda = .8$. In order to visualize this approach, figure 1 has been developed.

Figure 1 is a plot of $\psi$ versus d/h (frame depth/ shell thickness) for four $\lambda$ values. As implied by the abscissa scale which indicates relative frame strength, the frames are rectangular and only their depth is varied to vary strength. The basic geometries used in this figure are taken from a report by Trilling (19) and one of his curves (curve 2), as well as his experimental data (crosses) are plotted. The other three curves plotted on figure 1 represent the effect of changing $\lambda$. In all three cases, the distance between frames was the only parameter varied. Values of $\psi$ were obtained by using



1.30
1.20
1.10
1.00
.90
.80
.70
.60
.50
.40
.30
.20
PRESSURE FACTOR - ψ
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
FRAME DEPTH / SHELL THICKNESS - d/h
1
2
3
4
ψ vs d/h
ψ Determined using 792A
Rectangular Frames (19)
(+) λ = .84 - 1.0
1 λ = .974
2 λ = .898
3 λ = .814
4 λ = .683


FIGURE 1

von Sanden and Gunther's formula #92A to predict collapse pressure.

It will be noted that curve 3 of this figure lies almost entirely along the line, $\psi = 1.0$. The $\lambda$ value of this curve is .814, and it is this relationship which forms the basis of the design method cited above. Just as the earlier method of designing for a particular collapse mode proved inadequate, this method has also been unable to develop the efficient structures necessary to resist high pressures. The remainder of this section will be devoted to developing a design method which will result in highly efficient structures. To begin this development, a further look at figure 1 is necessary.

On the basis of curve 2 and the experimental points, one of Trilling's conclusions was that "except for very short thick tubes, . . . according to formula #92A the stronger the frame, the weaker the vessel." This was explained by noting that very strong frames form "hard spots" when, under external pressure, they contract less than the shell. The hard spots, in turn, cause stress concentrations in the otherwise uniformly contracting shell and lower collapse pressures.

Curve 4 indicates, however, that this conclusion is

not necessarily true, even for thin shells and large frames. The only difference between the model of curve 4 and Trilling's models is the decreased frame spacing which results in different $\lambda$ values; .898 for Tilling's models (curve 2) and .683 for curve 4. According to curve 4, shell strength actually increases with frame size when the frames are relatively close together. The seeming contradiction of Trilling may be explained by an extension of his logic. If the frames causing stress concentrations are closer together (low lambda) the overall stress pattern will be uniform enough that premature yielding and buckling will not take place.

The obvious lower limit of $\lambda$ is obtained when the frames are in contact ($\lambda \rightarrow 0$) (24) thereby creating a thicker shell (rectangular frames) or a webbed sandwich-type construction (T-frames). Both of these geometries result in putting the frame-caused stress concentrations in such close proximity that the overall stress pattern approaches uniformity. This uniformity of stress pattern implies that the yield strength of most of the material in the structure will be utilized before collapse occurs, which, in turn, implies that a structure of very high efficiency is possible in this $\lambda$ area.

With a lower limit of $\lambda$ established at 0, an upper

limit is necessary to define the range of geometries where the effect of increasing frame strength is not detrimental to collapse pressure. This limit will be referred to as $\lambda_m$. For structures with $\lambda$ values above $\lambda_m$, increasing the frame strength weakens the vessel. For $\lambda$ values below $\lambda_m$, increasing the frame strength strenthens the vessel. An initial value of this limit can be found by direct use of Windenburg's approximation of von Mises' formula for collapse between frames due to elastic shell instability. The development of this formula is outlined in Appendix A. In its simplest form, this formula is:

$$\psi = \frac{1.3}{\lambda^2}$$

Recalling that $\psi = \dfrac{p_c}{2\frac{h}{D_m}\sigma_y}$, Windenburg's formula determines instability collapse in terms of the hoop stress relation. i.e.,

$$p_c = 2\frac{h}{D_m}\sigma_y \frac{1.3}{\lambda^2}$$

therfore, when $\dfrac{1.3}{\lambda^2} = 1.0$, failure is due primarily to yielding of the material rather than plastic instability. At this point, $\lambda = 1.14$. If $\lambda$ 1.14, then instability collapse is indicated since $\psi$ 1.0. In this situation,

strengthening the frames cannot delay collapse. It may be tentatively assumed that when $\lambda$ 1.14, strengthening the frames increases overall strength. If this is so, then an initial approximation of $\lambda_m$ is 1.14. It is important to note that this is a very approximate method for determining $\lambda_m$. A glance of Trilling's results on figure 1 will indicate that collapse at $\psi$ values below 1.0 has occurred in structures with $\lambda$ values as low as .89. Also, with this method, $\lambda_m$ is independent of any individual structure parameters. Since this is questionable, another means of determining $\lambda_m$ is necessary.

Returning to curve 3 on figure 1, it will be recalled that for the $\lambda$ value used in that curve, $\psi = 1.0$ for all frame sizes. As long as the frames are of adequate strength or stiffness to carry a given percentage of the shell load, their size has no bearing on the strength of the cylinder. Figure 2 elaborates on this point. The curves of figure 2 are collapse pressures, determined by #92A, plotted as $\psi$ against values of $\lambda$. Each curve represents one size of frame and all structures have the same h/D ratio, so that only the distance between frames has been varied to obtain changing $\lambda$ values. The point at which all curves cross is at $\psi = 1.0$ and represents the afore-mentioned $\lambda_m$. Unlike the approximate Windenburg determination of this point, in this determination, $\lambda_m$

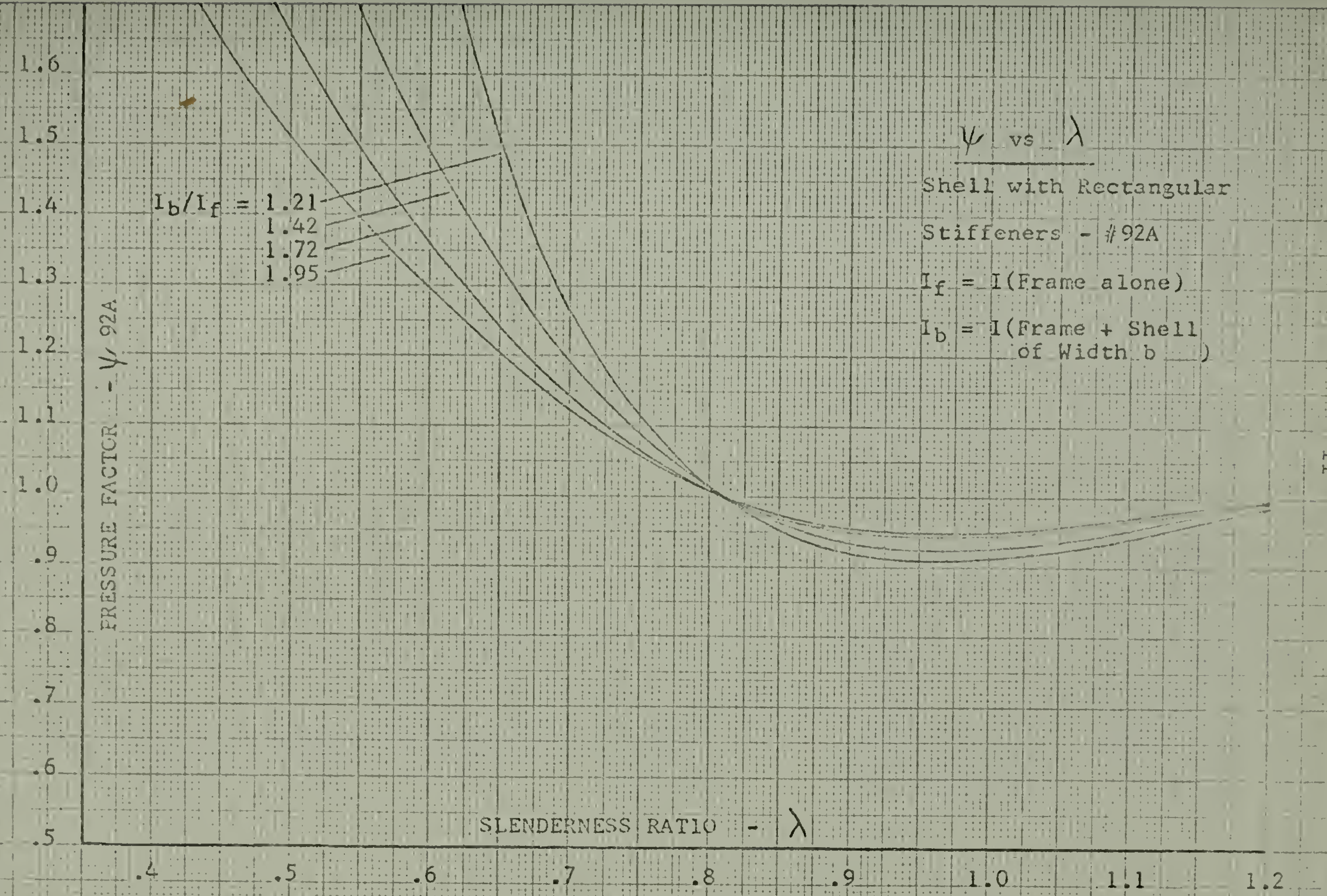

$\psi$ vs $\lambda$
Shell with Rectangular
Stiffeners - #92A
$I_f$ = I(Frame alone)
$I_b$ = I(Frame + Shell of Width b )
$I_b/I_f$ = 1.21
1.42
1.72
1.95
PRESSURE FACTOR - $\psi$, 92A
SLENDERNESS RATIO - $\lambda$
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2

FIGURE 2

depends on the value of h/D as well as the fact that $\psi = 1$. This dependence is borne out in the derivation of an expression for $\lambda_m$ from formula #92A. This derivation is found in Appendix E and results in the equation:

$$\lambda_m = \sqrt{\frac{2.055 D_m}{h} \quad \frac{\sigma_y}{E}} \tag{A}$$

which was used to plot $h/D_m$ versus $\lambda_m$ for different values of $\sigma_y$ on figure 3.

Figure 3 may be used to determine for a given $h/D_m$ and $\sigma_y$ what $\lambda$ is necessary to prevent collapse between the frames due to instability ($\psi < 1.0$). Areas to the right of the curve in question are areas of instability collapse, those to the left, primarily yield failure. As an example of this, Trilling's experimental results have again been plotted. Due to slightly varying geometries and $\sigma_y$'s, all the models he used fall within the areas of the dots shown. The figure indicates that for $\sigma_y$ = 30,000 psi (see appendix) all the models failed at a $\psi$ below 1.0. Reference to figure 1 shows that this was indeed the case,and a further glance at Reference (19) indicates that a number of Trilling's models failed by lobar buckling or general instability, rather than by yield initiated collapse.

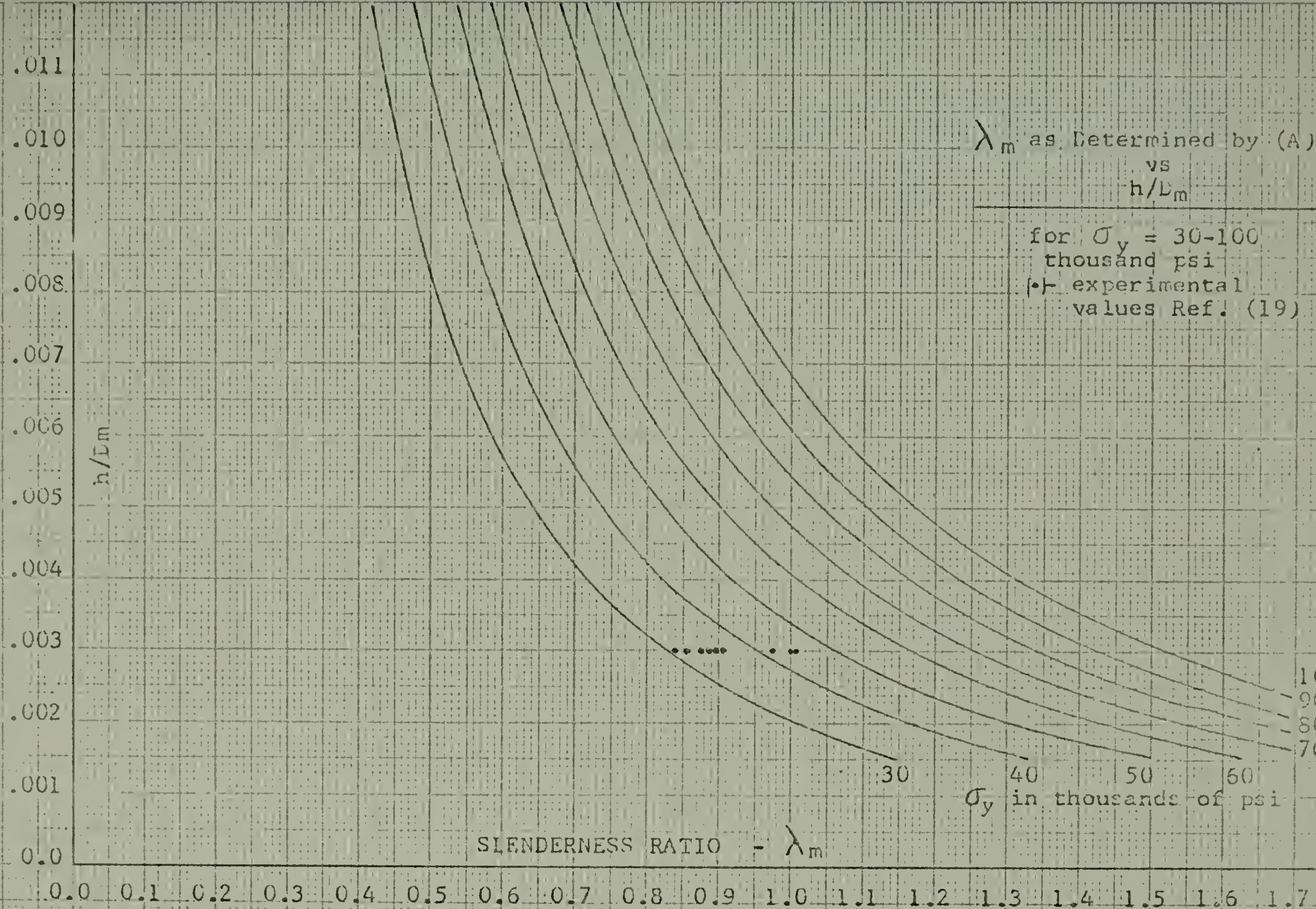

$\lambda_m$ as Determined by (A)
vs
$h/D_m$
for $\sigma_y$ = 30-100
thousand psi
(•) experimental
values Ref. (19)
.011
.010
.009
.008
.007
.006
.005
.004
.003
.002
.001
0.0
$h/D_m$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
SLENDERNESS RATIO - $\lambda_m$
100
90
80
70
30
40
50
60
$\sigma_y$ in thousands of psi

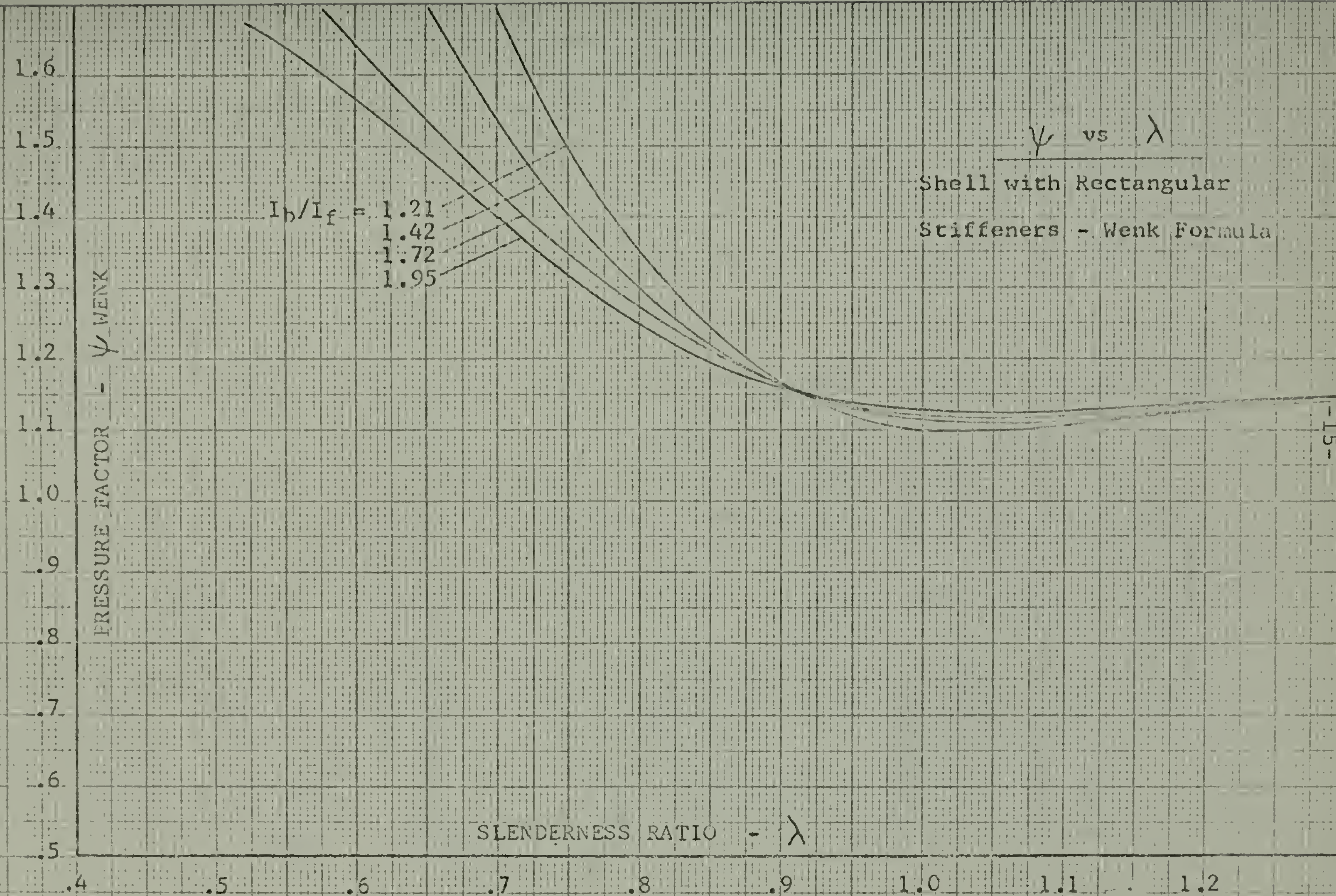
FIGURE 3

In Appendix A, a number of formulas and yield criterion are cited as being more accurate than the maximum circumferential stress criterion in determining collapse pressure due to yielding. One of these, Wenk's formula, has been investigated using the same models used in figures 1-3 in order to ascertain the influence of increased accuracy on the computation of $\lambda_m$. Figure 4 is the result of this investigation.

Wenk applied formulas #92 and #92A to the Hencky-von Mises yield criterion at mid-thickness, mid-bay, and obtained collapse pressures within nine percent of actual experimental values. The Hencky-von Mises criterion in two dimensions is:

$$\sigma_Y^2 = \sigma_T^2 - \sigma_T\sigma_F + \sigma_F^2$$

When it is applied to an unstiffened shell where $\sigma_T = 2\sigma_F$, yield failure occurs at $\sigma_T = 1.16\sigma_Y$, i.e., $\psi = 1.16$. Just as figure 2 indicated that for the maximum circumferential stress criterion, there is a $\lambda$ value where frame size has no effect on cylinder strength, figure 4 shows that the same is true for the Hencky-von Mises criterion. However, the $\lambda$ value in this case occurs when $\psi = 1.16$ instead of 1.0. Wenk's formula:



$\psi$ vs $\lambda$
Shell with Rectangular
Stiffeners - Wenk Formula
$I_b/I_f$ = 1.21
1.42
1.72
1.95
PRESSURE FACTOR - $\psi$ WENK
SLENDERNESS RATIO - $\lambda$
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2


FIGURE 4

$$p_c = \frac{2h}{D_m}\sigma_y \quad \frac{1}{\sqrt{F^2 - .5F + .25}}$$

like formula #92A, is actually the hoop stress relation times a factor(defined in Appendix E). When it is solved for F by setting $\psi = 1.16$, a $\lambda_m$ versus $h/D_m$ relation similar to formula (A) may be found. This formula is shown below and a detailed derivation of it is given in Appendix E:

$$\lambda_m = \sqrt{\frac{2.65\ D_m}{h} \quad \frac{\sigma_y}{E}} \qquad \text{(B)}$$

If plotted in the form of figure 3, this equation would result in a shift of all curves .11$\lambda$ to the right of those obtained from #92A. This shift reflects the conservative nature of von Sanden and Gunther's formula and criterion.

Thus far, equations and criteria have been developed to determine the extreme values of $\lambda$ between which stiffened cylindrical shell failure is primarily due to yielding of the material between the frames. Justification for using this as the design collapse mode comes from a number of sources. Wenk, a major advocate of the "one-hoss-shay" collapse philosophy, states that the approach of designing for yield failure is not inconsistent with that philosophy (22). Because of the effects of such

unknowns as material residual stresses and out of roundness, design for collapse by all three modes simultaneously is difficult, if not practically impossible. Therefore, the mode which is least affected by such strength modifiers should be the design collapse mode. Conversely, the collapse mode most affected by the modifiers should be the first mode taken into consideration and designed out of existance. Out of roundness is the primary reason for local failure of frames which can precipitate premature general instability collapse (15). For this reason, collapse by general instability has been noted as being of prime concern in the preliminary design (10).

Reference to formulas for general instability given in the appendix will reveal that this collapse mode, unlike the others, is a function of cylinder length (between bulkheads), as well as frame area, moment of inertia, and spacing. Judicious selection of bulkhead spacing (if a free variable) and minimum frame sizes can eliminate this collapse mode from consideration. This should be done in the first stages of design development. In the final stages collapse pressure for this mode should again be checked and out of roundness taken into consideration.

With general instability designed for, collapse between the frames due to buckling should be eliminated

by insuring that the final design $\lambda$ will be between 0 and $\lambda_m$. This means that the yield strength of the material will be utilized as fully as possible by insuring against premature failure initiated by instability. In this way, the benefits of using high strength materials will not be nullified by instability collapse at low pressures.

In the next section, considerations for the final step of designing for yield initiated failure between the frames will be developed. First, however, the question of frame adequacy must be looked into.

Figures 1, 2, 3, and 4 were all developed from analysis of cylindrical shells with rectangular stiffeners. T-shaped stiffeners are in general use today however, so the question may be raised as to what effect the shape of the stiffeners has. An investigation of the formulas for yield failure previously cited will reveal that they are all based on the assumption of adequate frame strength. The only frame parameters employed in them are flange width in contact with the shell and frame area. At $\lambda = \lambda_m$, even these parameters cease to influence collapse pressure. Therefore, it can be stated that frame shape has no influence on formulas (A) or (B). In general, as long as adequate strength is maintained, frame shape does not influence the collapse pressures of stiffened

cylindrical shells.

The question of what constitutes adequate frame strength has been directly approached by a number of investigators (2,11,19). The standard practice of von Sanden and Gunther (20) and other early designers was to design the frame to be able to withstand 1.1 times the total pressure on the length of plating it supported. This procedure, which employed the Foppl formula, was found to result in "excessively heavy frames" (19). In addition, the Foppl formula has been determined to give conservative results for frame buckling pressures. In light of these facts, the frame design requirement chosen for the purposes of this report was that the frame be strong enough to support no more than the load on one effective length of shell. The conservativeness of the Foppl formula was accounted for by assuming that it predicted frames alternatively 4, 5, or 6 times too strong. The resulting formula used is:

$$p_c \text{ frame} = p_c \text{ shell} = \frac{24EI_b}{D_b{}^3L_i} \times (4,5,\text{or } 6)$$

Before proceeding into the actual design iteration resulting from the establishment of design criteria for stiffened cyl.ndrical pressure vessels under hydrostatic pressure, a recapitulation of the criteria is in order:

1. Upon definition of operating requirements, initial Structure parameters should be established to minimize the possibility of general instability collapse of frame and shell.

2. Possibility of collapse due to instability between frames should be reduced by limiting design choices to geometries located between $\lambda = 0$ and $\lambda = \lambda_m$.

## III. VESSEL EFFICIENCY

The measure of efficiency used in this paper was suggested by Wenk in (22). It is slightly more complicated than the traditional Weight/Displacement ratio in that it takes into account collapse pressure.

$$\eta = \frac{p_c V}{10^5 W}$$

$p_c$ is the collapse pressure - lbs/in$^2$

V is the volume per unit length of cylinder - in$^3$/ft length

W is weight per unit length of cylinder - lbs/ft length

When used in a qualitative manner, this term clearly shows the relative efficiencies of different geometries. However, it is not independent of material properties such as density and yield stress. The efficiencies in this report have been calculated using HY 80 as the material. In order to apply them to materials of other densities and yield strengths, they must be multiplied by the following factors:

$$\frac{\sigma y}{80,000} \quad \text{and/or} \quad \frac{.284}{\rho}$$

For example, a geometry which would have an efficiency of 1.25 for HY 80, has an efficiency of:

$$1.25 * \frac{50,000}{80,000} * \frac{.284}{.1} = 2.19$$

when the material is 50,000 psi aluminum. Once found in the above manner, the efficiency may be used to calculate the traditional Wt/Displacement ratio using the following conversion formula:

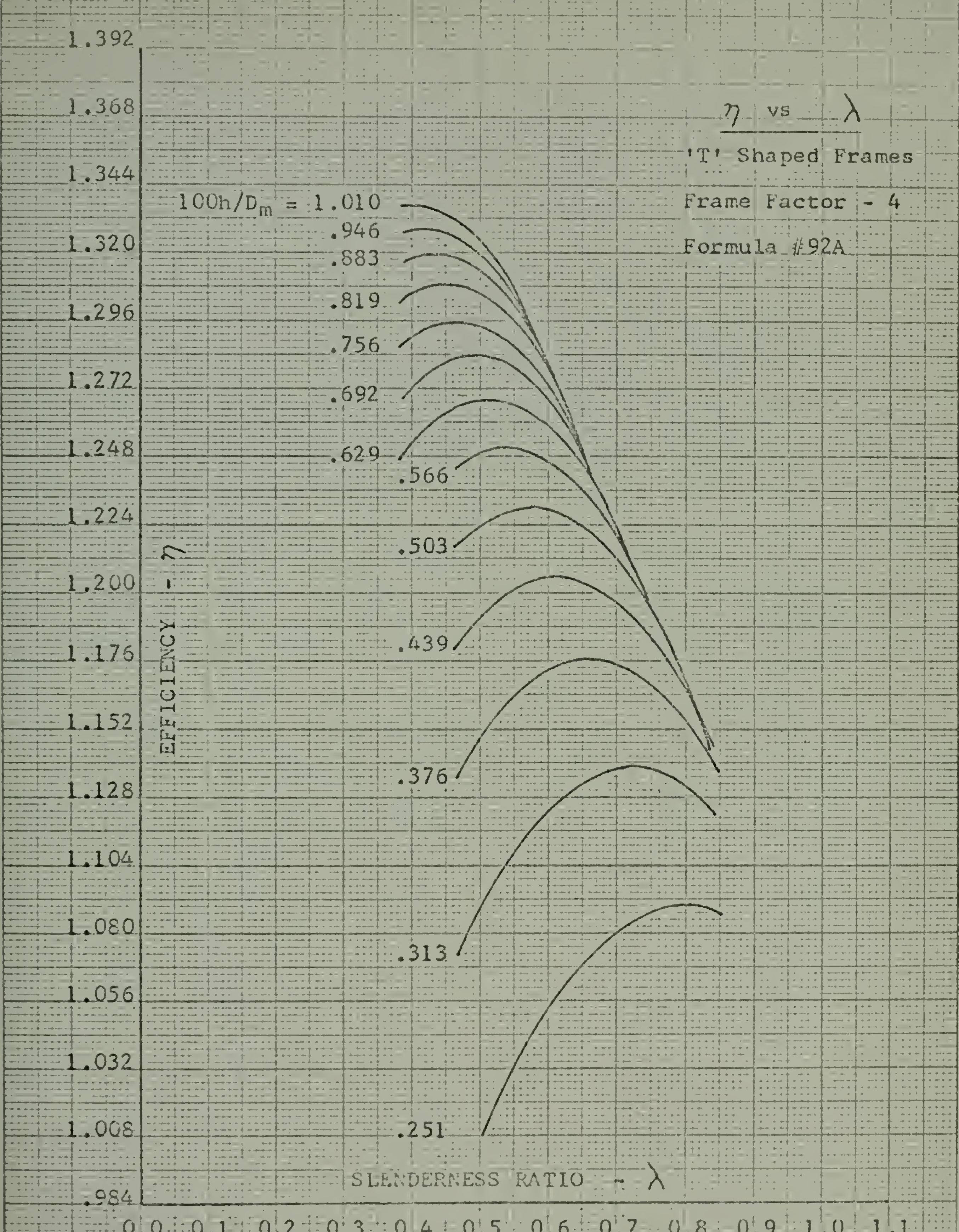
$$\frac{Wt}{\Delta} = \frac{p_c}{3720\eta}$$

Reasons for this manipulation of $\eta$ will become more clear at the end of the next section.

## IV. DESIGN ITERATION

In the preceding section, design criteria have been developed which, if used, will produce a generalized cylinder geometry whose $\lambda$ value will be between 0 and $\lambda_m$. This will result in a collapse pressure factor in excess of 1.0, thus insuring that collapse will be primarily due to yield and the yield strength of the material will be highly utilized. This section will outline development of an iterative method to find the particular $\lambda$ and $\psi$ values which result in maximum structural efficiency for a given collapse pressure.

Figures 5, 6, and 7 are plots of efficiency against $\lambda$ for different values of $h/D_m$ and frame strength factors. As stated previously, Foppl's frame formula has been used in a modified form which arbitrarily reduces its apparent conservativeness. In determining frame size for figure 5, Foppl's formula was multiplied by a factor of 4. In figure 6, the frame factor was 5 and in figure 7, it was 6. The effect of this increasing theoretical strength of frames was to reduce frame size and increase apparent structure efficiency in succeeding figures. Since this is an arbitrary change, model tests are necessary to determine the actual frame factor which should be used. It was felt that a range of factors such as that covered



1.392
1.368
1.344
1.320
1.296
1.272
1.248
1.224
1.200
1.176
1.152
1.128
1.104
1.080
1.056
1.032
1.008
.984
$\eta$ vs $\lambda$
'T' Shaped Frames
Frame Factor - 4
Formula #92A
$100h/D_m$ = 1.010
.946
.883
.819
.756
.692
.629
.566
.503
.439
.376
.313
.251
EFFICIENCY - $\eta$
SLENDERNESS RATIO - $\lambda$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1


FIGURE 5

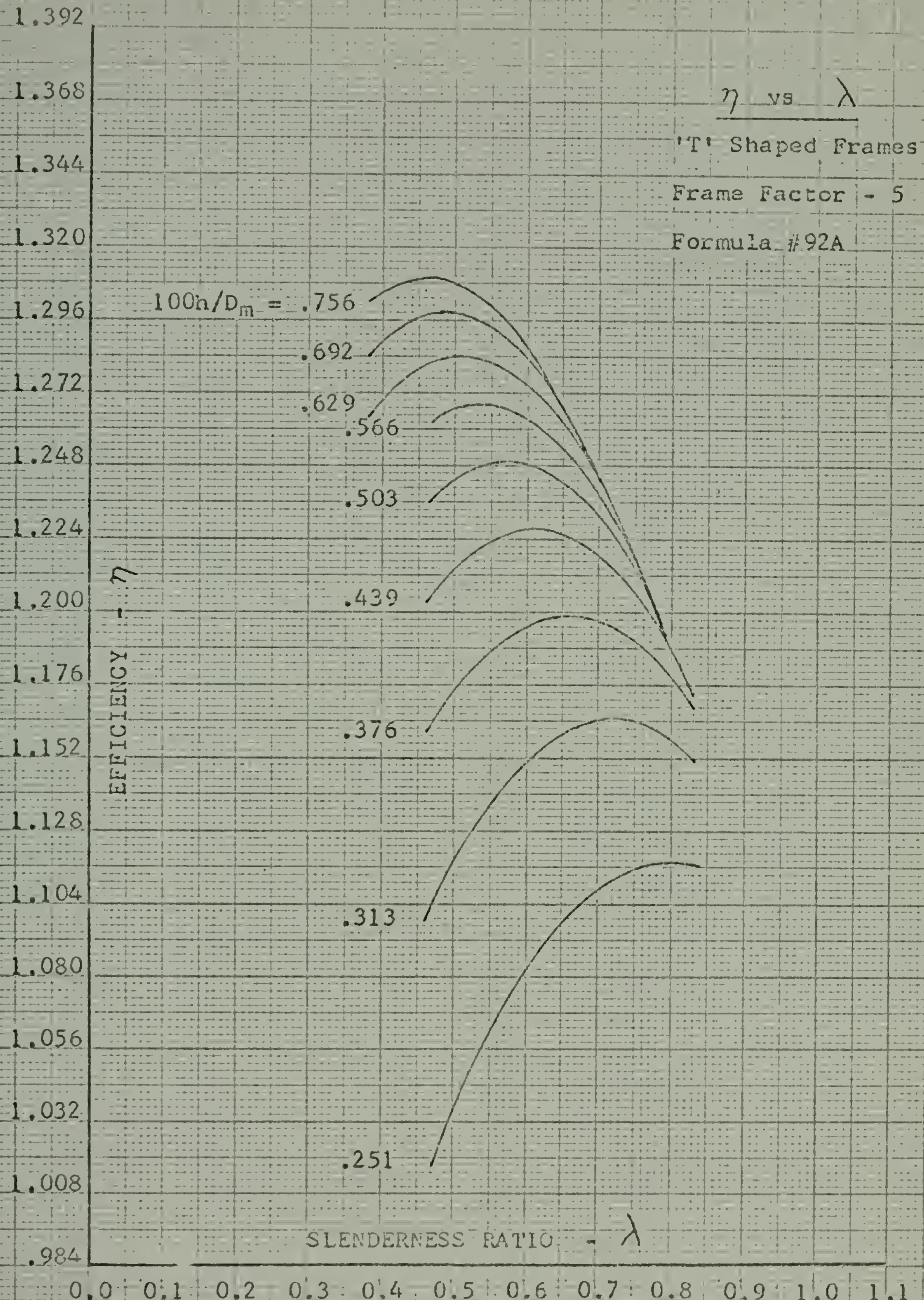
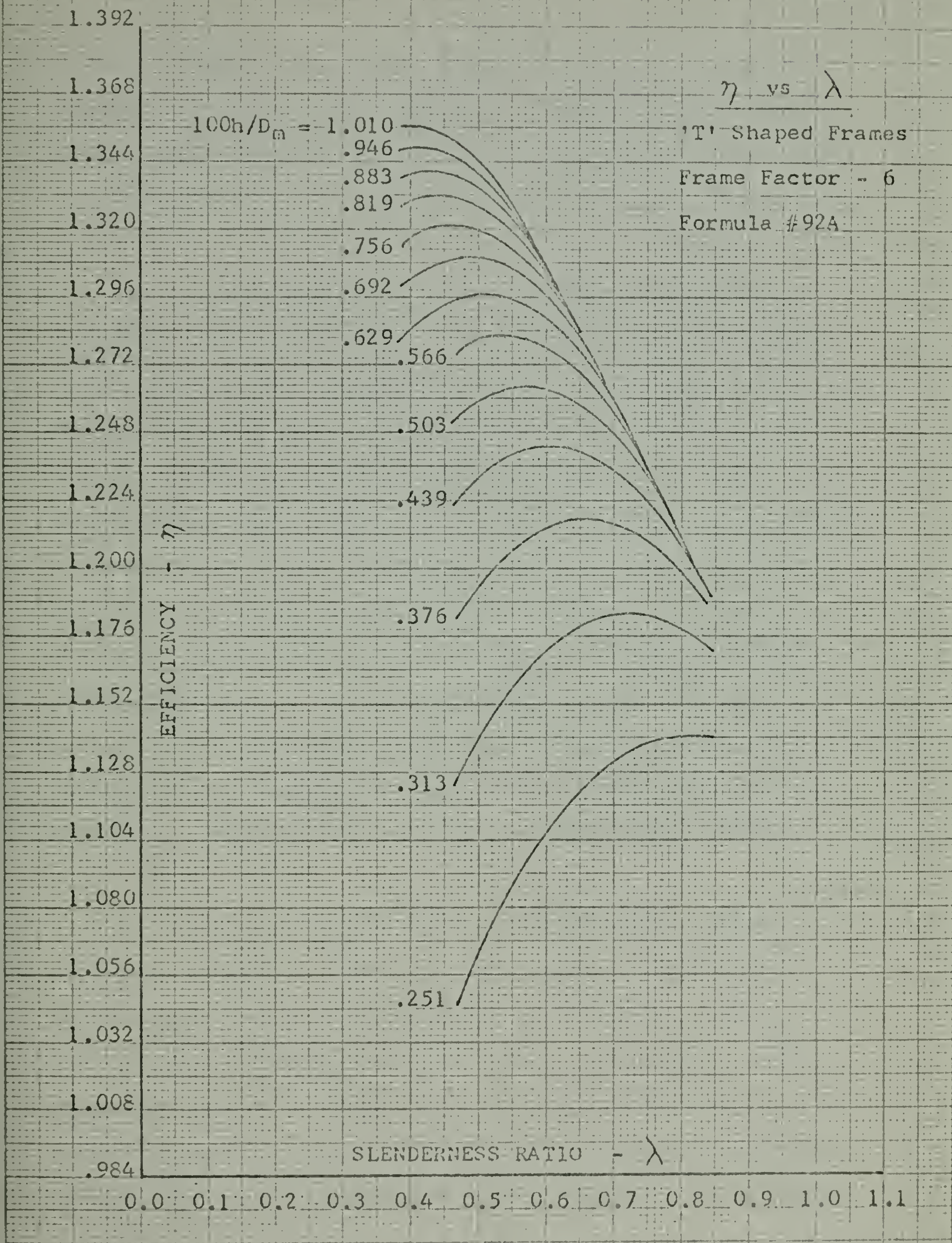

1.392
1.368
1.344
1.320
1.296
1.272
1.248
1.224
1.200
1.176
1.152
1.128
1.104
1.080
1.056
1.032
1.008
.984
$\eta$ vs $\lambda$
'T' Shaped Frames
Frame Factor - 5
Formula #92A
$100h/D_m$ = .756
.692
.629
.566
.503
.439
.376
.313
.251
EFFICIENCY - $\eta$
SLENDERNESS RATIO - $\lambda$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1

FIGURE 6



$\eta$ vs $\lambda$
'T' Shaped Frames
Frame Factor - 6
Formula #92A
100h/$D_m$ = 1.010
.946
.883
.819
.756
.692
.629
.566
.503
.439
.376
.313
.251
EFFICIENCY - $\eta$
SLENDERNESS RATIO - $\lambda$
1.392
1.368
1.344
1.320
1.296
1.272
1.248
1.224
1.200
1.176
1.152
1.128
1.104
1.080
1.056
1.032
1.008
.984
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1


FIGURE 7

would indicate design trends which may be beneficial in general use without the necessity of establishing rigorous theoretical frame design criteria. Further substantiation of this method will be provided later in this section.

The formula used to determine collapse pressures for these three figures was #92A (maximum circumferential stress criterion). Although this is now considered to be less accurate than a number of possible formulas, it has the advantages of always being conservative and having been widely used. The curves in figures 5, 6, and 7 indicate that for a given $h/D_m$ and frame factor, the efficiency increases with $\lambda$ until a maximum, $\eta_{max}$, is reached. Thereafter, the efficiency steadily decreases with increasing $\lambda$ . It is this point, $\eta_{max}$, that the designer of stiffened cylindrical pressure vessels is interested in, therefore, only enough points have been plotted to indicate the hump of each efficiency curve.

An interesting trend which is readily apparent in all three figures is that the efficiencies of all h/D combinations lie on the same line at the point where $\lambda = \lambda_m$ for each $h/D_m$ (see figure 3). In figure 5, this line has a slope of about -.5. In subsequent figures, the slopes appear the same, so, since the lines begin at higher values, the final efficiencies will be higher. Of course,

extension of the lines upward to maximum efficiency also requires extension to the left to $\lambda = 0$ and **extension** of the series of h/D curves to extremely thick cylinders. This extrapolation of computed data can be justified on the grounds that only very short, thick cylinders fail by pure yielding. Failure in this way takes the fullest possible advantage of material yield strength and results in the highest possible structural efficiency. Since efficiency in this report is also a function of displacement to weight ratio, the veracity of that previous statement, in the general sense, is dependent on material density.

Another point which is illustrated in figures 5-7, is that for each $h/D_m$ there is only one value of $\lambda$ at which maximum efficincy is reached, regardless of frame factor. The effect of the frame factor is to change the actual maximum efficiency reached, but it has no effect on the $\lambda$ value where it is reached. This direct relationship between $h/D_m$ and $\lambda$ at maximum efficiency is shown in figure 8. It may be explained in part by the fact that neither $\lambda$ nor $h/D_m$ contain any frame size parameters and both Wenk's formula and formula #92A assume adequate frame strength. Also at that point of maximum efficiency, there are discrete values of $\psi$ which are dependent on frame factor. In order to show the relationship of frame

factor, $\lambda$ and $\psi$ at maximum efficiency, figures 9 and 10 have been plotted.

Before entering into the use of these figures, however, a slight digression is necessary. Recalling that formula #92A, used to obtain figures 5-7, was shown to yield less accurate collapse pressures than other formulas, it became necessary to investigate a formula of higher accuracy to determine if the same trends still held. The formula used was developed by Wenk and has been introduced in Section II. Data similar to that shown in figures 5, 6, and 7, was developed for the same geometries using Wenk's formula. The actual results are given in Appendix G. Integrated curves from these results have been drawn on figures 8 and 10.

Figure 8 is a graph of h/D versus $\lambda$ for points of maximum efficiency. The two curves represent results of the use of formula #92A and Wenk's formula. It will be noted that Wenk's formula shifts the curve to the right. Since this expression has been known to predict collapse pressures in excess of actual values, it may be expected that the curve for actual structures will lie between the curves shown. No mention of frame size is made on this graph, as in neither formula did frame strength alter the relationship of h/D and $\lambda$ at maximum efficiency.

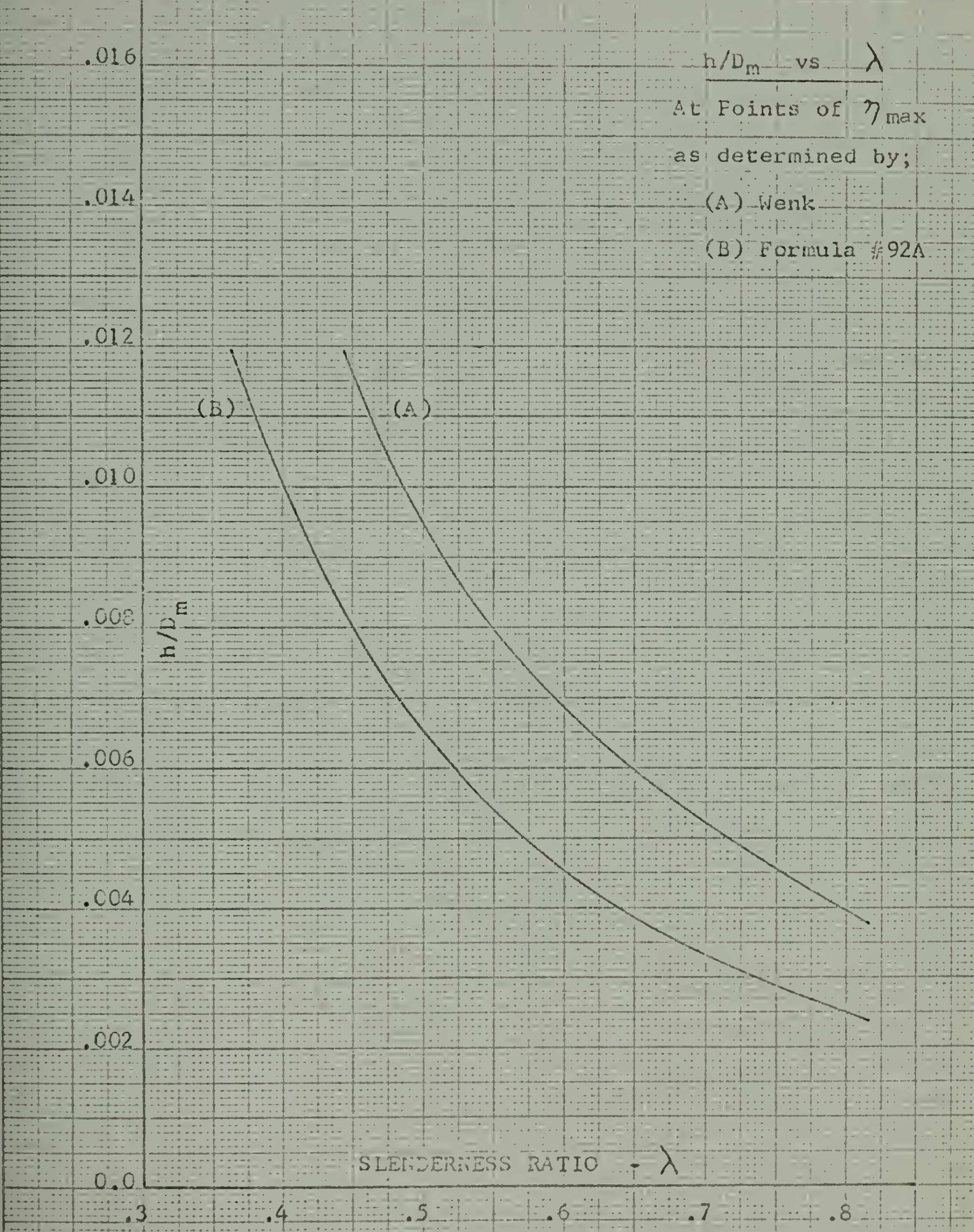

h/D_m vs λ
At Points of η_max
as determined by;
(A) Wenk
(B) Formula #92A
.016
.014
.012
.010
.008
.006
.004
.002
0.0
h/D_m
(B)
(A)
SLENDERNESS RATIO - λ
.3
.4
.5
.6
.7
.8

FIGURE 8

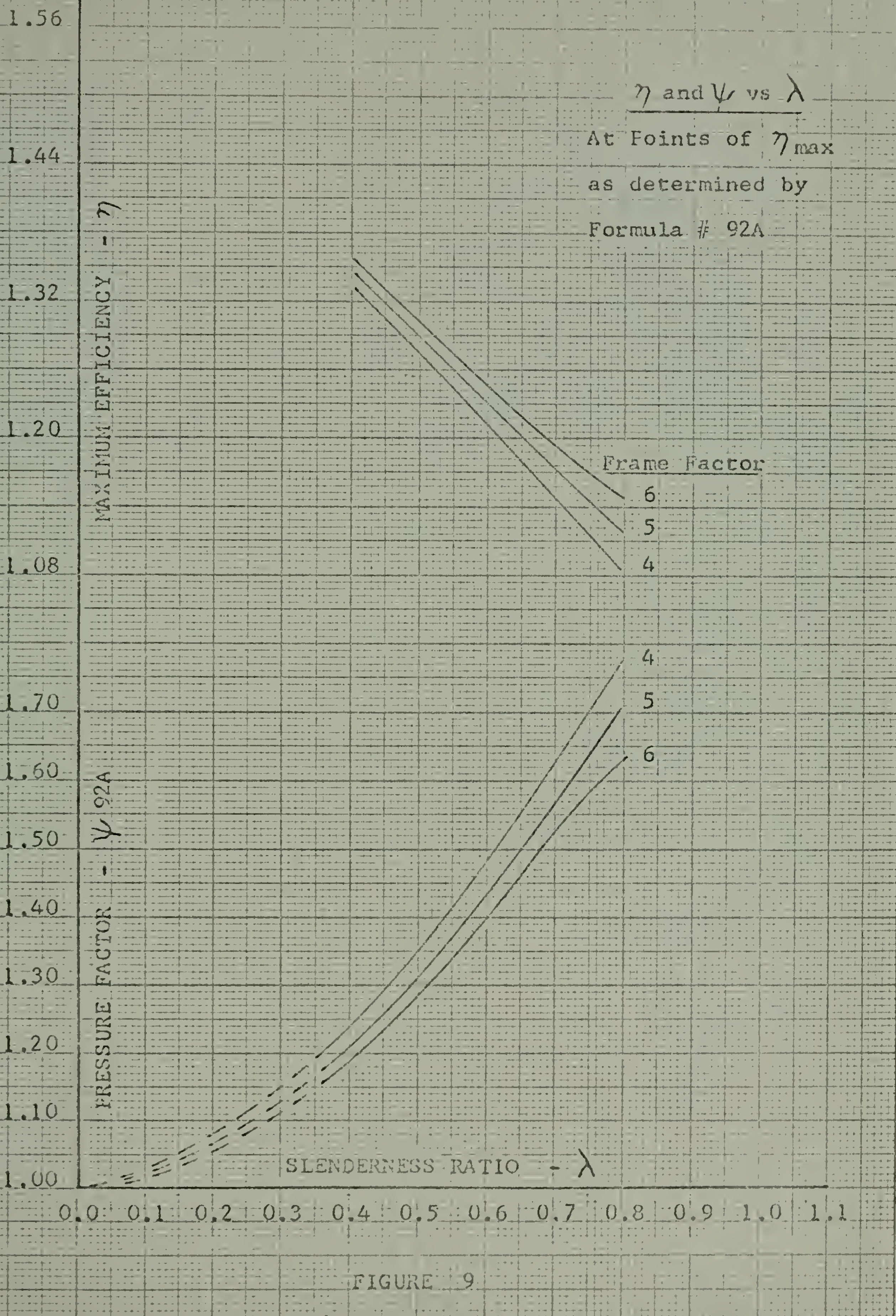

$\eta$ and $\psi$ vs $\lambda$
At Points of $\eta_{max}$
as determined by
Formula # 92A
MAXIMUM EFFICIENCY - $\eta$
1.56
1.44
1.32
1.20
1.08
Frame Factor
6
5
4
4
5
6
PRESSURE FACTOR - $\psi$ 92A
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
SLENDERNESS RATIO - $\lambda$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1

FIGURE 9

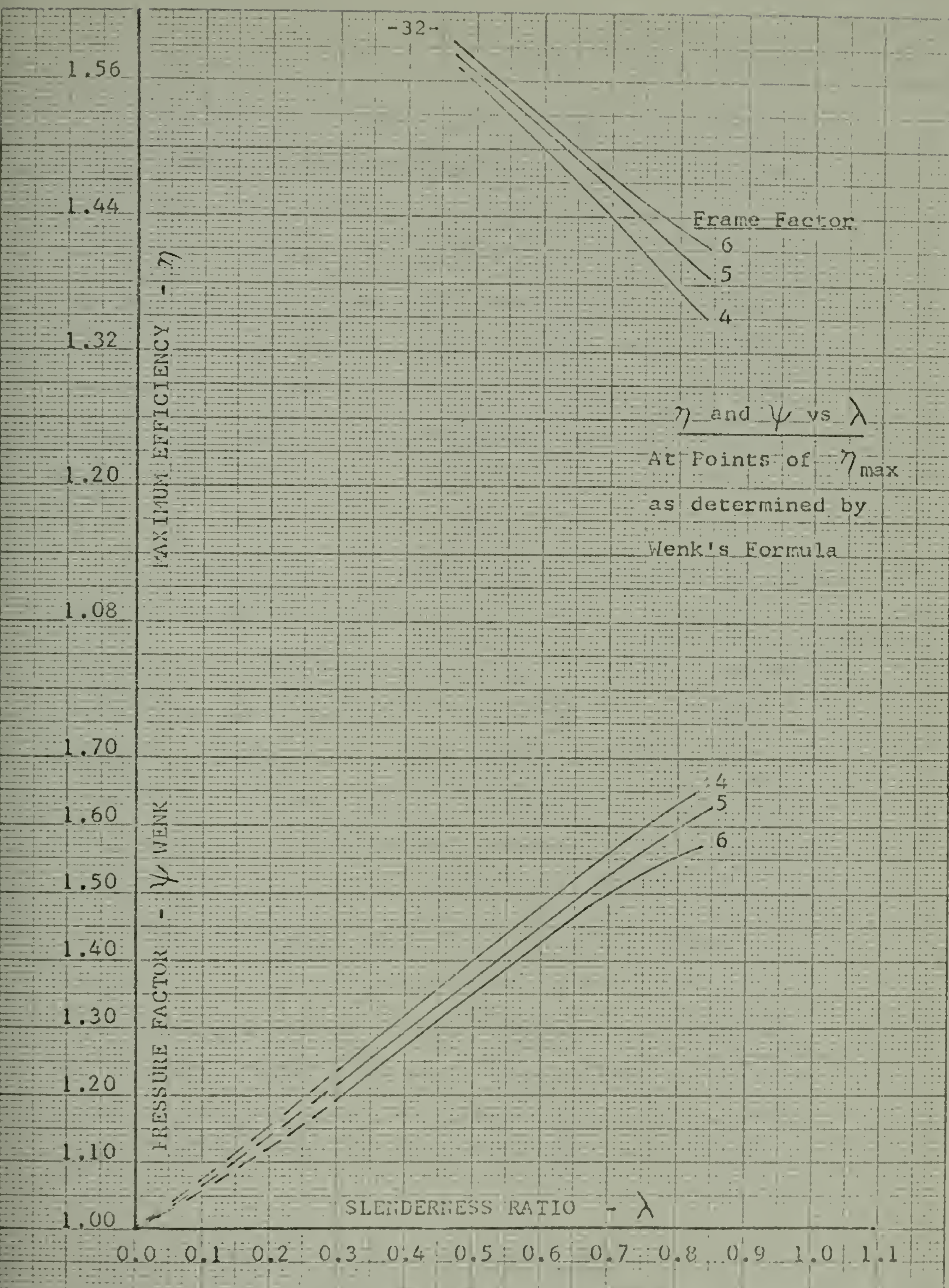

1.56
1.44
1.32
1.20
1.08
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
MAXIMUM EFFICIENCY - $\eta$
PRESSURE FACTOR - $\psi$ WENK
Frame Factor
6
5
4
$\eta$ and $\psi$ vs $\lambda$
At Points of $\eta_{max}$
as determined by
Wenk's Formula
4
5
6
SLENDERNESS RATIO - $\lambda$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1

FIGURE 10

Figures 9 and 10 are graphs of $\eta_{max}$, $\lambda$ and $\psi$ for #92A and Wenk's formula respectively. The three lines in each case indicate the three frame factors used. Comparison of these figures shows that increased accuracy does not change the trends previously noted. In fact, the $\eta$ versus $\lambda$ curves are exactly alike. The 25 percent increase in efficiencies in figure 10 over figure 9 is another reflection of the conservativeness of formula #92A.

With development of figures 8-10 complete, an iterative method to determine geometries of maximum efficiency using these figures can be outlined. Geometric parameters which can be found are $h/D_m$ and $\lambda$. Given material properties, distance between stiffeners may be computed. Actual stiffener parameters are left up to the discretion of the designer as the only requirement of the formulas used in this procedure is that the stiffeners be of adequate strength and stiffness. For comparison purposes, the frame parameters used in this report and those frame dimensional ratios coinciding with maximum efficiency structures are given in Appendix C. To begin the iterative procedure, the information which must be specified is:

1. type of analysis to be used
   a. maximum principle stress, #92A

b. Hencky-von Mises stress at mid-thickness, mid-bay, #92 and #92A

2. frame factor to be used: 4, 5, or 6

3. ultimate collapse pressure desired

4. material properties, $\sigma_y$ and E (for these curves $E = 3 \times 10^7$)

5. outside diameter

An example of the procedure is given on the next page.

Example 1:

Given: Use of #92A, Frame Factor - 5

$\sigma_y$ = 80,000 psi

D = 40 ft.

$p_c$ = 1000 psi

Steps:

1. Initial $h/D_m$ guess from hoop stress

$p_c = 2(h/D)\sigma_y \qquad h/D = h/D_m$

$(h/D_m) = 1000/(2*80,000)$

$(h/D_m) = .00692$

$\lambda$ = .48 from figure 10

$\psi$ = 1.29 from figure 11

$$\psi = \frac{p_c}{2\frac{h}{D}\sigma_y} = 1.29$$

$\therefore\ p_c$ = 1290 psi - which is too high

2. Reduce $h/D_m$ by division

$p_{c1} = \frac{1000}{1.29} = 775$ psi

$(h/D_m) = 775/(2*80,000)$

$(h/D_m) = .00484$

$\lambda = .58$

$\psi = 1.41$

$p_c$ = 1.41*775 = 1090 psi - which is still too high

3. Reduce $h/D_m$ again

$p_{c2}$ = $\frac{775}{1.09}$ = 710 psi

$(h/D_m)$ = $710/(2*80{,}000)$ = .00443

$\lambda$ = .606

$\psi$ = 1.44

$p_c$ = $1.44*710$ = 1020 psi - which is close enough, but try a 4th iteration

4. Reduce $h/D_m$

$p_{c3}$ = $\frac{710}{1.02}$ = 697 psi

$(h/D_m)$ = $697/(2*80{,}000)$ = .00436

$\lambda$ = .613

$\psi$ = 1.45

$p_c$ = $1.45*697$ = 1012 psi

Answer:

$\lambda$ = .613

$h/D$ = .00436

$\eta$ = 1.22

$\psi$ = 1.45

Results:

D = 40 ft.

h = 2.1 in.

$L_f$ = 1.56 ft.

Wt. = 18,000 lb/ft

conventional Wt/Disp = .222

## V. DISCUSSION OF RESULTS

The results of this report may be divided into two classes; those due to analysis and those due to synthesis. Analysis by formula #92A and Wenk's Formula at low lambda values has shown that considerable gains in efficiency can be realized by placing frames close together and reducing shell thickness. This results in $\psi$ values greater than 1 and helps to realize the stress potentials of high strength materials. The extrapolation of the design curves in figures 11 and 12 appears to justify the use of sandwich construction for very high pressures. Such construction is analogous to design in the very low lambda region, and a discussion of this is included in the appendix. Inferences which may be drawn from the treatment of very low lambda design in Appendix B concern the apparent inapplicability of $\lambda$ and $\psi$ in this region. This results from the fact that in this region, frame strength contributes so much to overall strength that it cannot be ignored as it is in both $\lambda$ and $\psi$ .

At the high end of the low lambda region, maximum structural efficiency can be realized only in relatively low pressure vessels. This is shown in two examples in the appendix.

The result of synthesis in this report is the iterative method outlined in Section IV. By its very nature, this method results in thin-skinned vessels with frames relatively close together. The questions which this geometry may raise concerning modes of collapse other than yield-initiated, must be answered by independent analyses. It will be noted, however, that the geometries used to plot the curves of figures 8-13 were analyzed for failure both by instability between frames and general instability and the indicated collapse mode for all was yield-initiated failure between rings.

## VI. CONCLUSIONS

1. The dimensionless parameter, $\lambda$, may be used to define all stiffened cylindrical pressure vessel geometries except those in which the frames provide little support because of their wide spacing, and those vessels in which the frames provide most of the strength by virtue of their size and close spacing.

2. Absolute structure efficiencies in the low lambda region are highly dependent on frame spacing and only slightly dependent on frame size (as long as the frames are of adequate strength).

3. Relative structure efficiencies in the low lambda region are virtually independent of frame size. The lambda values at which maximum efficiency is reached depend on the $h/D_m$ ratio and are not affected by changes in frame size.

4. Structures with $\psi$ values well in excess of 1.0 are reasonable and generally provide increased efficiencies over those where $\psi$ is limited to 1.0.

5. As the design collapse pressure increases, the value of lambda which coincides with maximum structural efficiency decreases. This implies that higher efficiencies can be obtained by reducing shell thickness well below that required by the hoop stress relation and simultaneously increasing the number of frames.

6. At extremely low lambda values, general instability becomes the most likely mode of failure.

## VII. RECOMMENDATIONS

Since this thesis was limited to studying the parameter, $\lambda$, and its effect on efficiency, other definitive parameters, such as those now used at the David Taylor Model Basin should be investigated for their effect on efficiency. It has been noted that the lambda term is relatively useless when the frames are very close together and are very strong. This condition should be corrected by defining a new parameter similar to lambda in its application, but allowing for the effect of frame strength in geometric comparisons.

The basic approach of this thesis should be used to extend the curves of maximum efficiencies, found in Section IV, into the regions of higher collpase pressures. With this extention, a thorough re-examination of general instability collapse in these regions should be undertaken. Finally, experimental tests of maximum efficiency geometries, as developed by the design iteration, should be undertaken to provide evidence of the method's worth.

# REFERENCES


1. Boichot, L. and Reynolds, T.E., "Inelastic Buckling Tests on Ring Stiffened Cylinders Under Hydrostatic Pressure", DTMB Report 1992, May 1965.

2. Evans, J.H. and Adamchak, J.C., Ocean Engineering Structures, Chapter on Shell Analysis and Design, M.I.T. Press, 1969.

3. Gerard, G., "Minimum Weight Design of Ring Stiffened Cylinders Under External Pressure", Journal of Ship Research, Vol. 5, No. 2, Sept. 1961, pp. 44-49.

4. Heller, S.R. and Dunham, F.W., "Comparative Behavior of Submarine Pressure Hull Structure of Different Scales Under Uniform External Pressure", ASNE Journal, May 1963, p. 397.

5. Krenzke, M.A. and Reynolds, T.E., "Structural Research on Submarine Pressure Hulls at the David Taylor Model Basin", Journal of Hydronautics, Vol. 1, No. 1, July 1967, p. 27.

6. Krenzke, M.A. and Kiernan, T.J., "Structural Development of a Titanium Oceanographic Vehicle for Operating Depths of 15,000 to 20,000 Feet", DTMB Report 1677, Sept. 1963.

7. Krenzke and Pulos, Discussion, Reference 18.

8. Lunchick, M.E. and Overby, J.A., "An Experimental Investigation of the Yield Strength of a Machined Ring Stiffened Cylindrical Shell (Model BR-7M) Under External Hydrostatic Pressure", DTMB Report 1255, Nov. 1958.

9. Lunchick, M.E., "Yield Failure of Stiffened Cylinders Under Hydrostatic Pressure", DTMB Report 1291, Jan. 1959.

10. Lunchick, M.E., "Plastic Axisymmetric Buckling of Ring Stiffened Cylindrical Shells Fabricated From Strain Hardening Materials and Subjected to External Hydrostatic Pressure", DTMB Report 1393, Jan. 1961.

11. McGinley, E.S. II, "Optimization of Ring Stiffened Cylindrical Shells for Practical Hydrospace Applications", Thesis, N.A., M.I.T., 1970.

12. MacNaught, D.F., "Submarine Pressure Hull Design", Principles of Naval Architecture, Comstock, J.P., 1967 SNAME, Chap. 4, Sec. 8.

13. Pulos, J.G. and Salerno, V.L., "Axisymmetric Elastic Deformations and Stresses in a Ring Stiffened, Perfectly Circular Cylindrical Shell Under External Hydrostatic Pressure", DTMB Report 1497, Sept. 1961.

14. Pulos, J.G., "Structural Analysis and Design Considerations for Cylindrical Pressure Hulls", DTMB Report 1639, April 1963.

15. Pulos, J.G. and Krenzke, M.A., "Recent Developments in Pressure Hull Structures and Materials for Hydro-Space Vehicles", DTMB Report 2137, Dec. 1965.

16. Reynolds, T.E., "A Graphical Method for Determining the General Instability Strength of Stiffened Cylindrical Shells", DTMB Report 1106, Sept. 1957.

17. Salerno, V.L. and Pulos, J.G., "Stress Distribution in a Circular Cylindrical Shell Under Hydrostatic Pressure Supported by Equally Spaced Circular Ring Frames", PIBAL Report 210, Dec. 1952.

18. Saunders, H.E. and Windenburg, D.F., "The Use of Models in Determining the Strength of Thin-Walled Structures", ASME Transactions, Vol 54, 1932, pp. 263-275.

19. Trilling, C., "The Influence of Stiffening Rings on the Strength of Thin Cylindrical Shells Under External Pressure", Experimental Model Basin Report 396, Feb. 1935.

20. von Sanden, K. and Gunther, K., "The Strength of Cylindrical Shells, Stiffened by Frames and Bulkheads Under Uniform External Pressure On All Sides", DTMB Translation 38, March 1952.

21. Wenk, E., Jr., Stark, R.E. and Peugh, D.G., "Tests of the Yield Strength of Ring Stiffened Cylindrical Shells, Models BR-2 and 2A, Subjected to Hydrostatic Pressure", DTMB Report C-440, Feb. 1954.

22. Wenk, E., "Pressure Vessel Analysis of Submarine Hulls", Supplement to the Welding Journal, Jan. 1961.

23. Wenk, E., Jr., "Feasibility of Pressure Hulls for Ultradeep Running Submarines", Journal of Engineering for Industry, Aug. 1962.

24. Windenburg, D.F. and Saunders, H.E., "Strength of Thin Cylindrical Shells Under External Pressure", ASME Transactions, Vol 53, 1931, Pp. 207-218.

25. Windenburg, D.F. and Trilling, C., "Collapse by Instability of Thin Cylindrical Shells Under External Pressure", ASME Transactions, Vol 56, 1934, Pp. 819-825.

# APPENDIX A

## HISTORICAL BACKGROUND

Prior to 1917, the design of stiffened cylindrical pressure vessels was based on the assumption that vessel strength depended primarily on frame strength. The shell was treated as a flange of the frame and strength was calculated from equations which defined collapse by instability of frame rings under radial loads. A widely used formula for this was developed by Levy, but is commonly called Foppl's Formula (19,24):

$$q_f = \frac{3EI}{R_f^{\,3}}$$

While this formula was relatively accurate for the time, and where frames alone were involved, when it was applied to a stiffened cylindrical shell, its shortcomings became apparent as higher collapse pressures were required.

In order to improve upon this frame-oriented theory, von Sanden and Gunther derived expressions based on the observation that stiffened cylindrical shells were not collections of connected rings, but, in fact, were closed cylinders reinforced by ring frames. Deciding that the shell was the real strength member, they predicted failure due to shell yielding at the area of maximum stress. This

maximum principle stress criterion could be satisfied by the longitudinal stress at the frames or by the circumferential stress midway between them. Formulas for both stresses were developed and the well-known formulas #92 and #92A resulted when pressures were for in terms of the stresses. Substituting material yield stresses into #92 and #92A resulted in two yield pressures for the structure, of which the lower one was said to predict failure. It must be emphasized that failure was predicted on the basis of the onset of yielding.

For a number of years after the 1918 publication of these formulas, #92 was thought to give the critical collapse pressure since the greater stress was longitudinal. In 1935, however, this assumption was found to be in error when circumferential stress was determined to be critical in the collapse of thin-shelled pressure vessels (19).

The von Sanden and Gunther formulas are:

#92 $$p_y = \frac{2^h/D_m \sigma_y}{\frac{1}{2} + 1.81K \left(\frac{.85 - B}{1 + \beta}\right)}$$

$$\#92A \qquad p_y = \frac{2^h/D_m\sigma_y}{\frac{1}{2} + H\left(\frac{.85 - B}{1 + \beta}\right)}$$

The variables K, H, B, and β are defined in the Formula section.

About this same time, it was generally recognized that the area between stiffeners very rarely collapsed as a result of pure yielding. In anything but thick cylinders, collapse occurred from a combination of yielding and buckling when the distance between stiffeners (effective length) was below a critical length. This critical length was dependent on shell L/D and h/D ratios as well as material properties. When the distance between stiffeners was above the critical length, the tube was considered to be of infinite length and the formula of Breese and Bryan for collapse by instability could be applied:

$$p_c = \frac{2E}{1 - \mu^2} (h/D_m)^3$$

For cylinders below the critical length, formulas for collapse by instability were developed by von Mises and Southwell. Windenburg simplified von Mises lengthy

formula and, by sacrificing a very small degree of accuracy, came up with the following formula (25):

$$p_c = \frac{2.42E}{(1-\mu^2)^{3/4}} \quad \frac{(h/D)^{5/2}}{[L/D_m - .45(h/D_m)\ ]^{1/2}}$$

At the time Windenburg developed the above formula, a $p_c$ versus L/D coordinate system was commonly used to plot experimental and theoretical results for changing values of h/D. In an effort to develop an analog to the slenderness ratio, L/r, in column theory, and aid comparison of results, he developed a new coordinate system based on the coordinates:

$$\psi_0 \doteq \frac{p}{2h/D_m}$$

$$\lambda_0 = \sqrt[4]{\frac{(L/D_m)^2}{(100h/D_m)^3}}$$

Windenburg then corrected his coordinates for the physical properties of materials in the following manner and came up with the well-known variabels, slenderness ratio $\lambda$ , and pressure factor, $\psi$ .

$$\psi = \frac{\psi_0}{\sigma_y} = \frac{p_c}{2h/D_m\sigma_y}$$

$$\lambda = \lambda_0 \sqrt{\left(\frac{1000\sigma_y}{E}\right)\left(\frac{1 - \mu^2}{.91}\right)^{3/4}} = \sqrt[4]{\frac{(L/D_m)^2}{(h/D_m)^3}}\sqrt{\frac{\sigma y}{E}}$$

This done, Windenburg rewrote his instability formula in terms of his new variables:

$$\psi = \frac{1.30}{\lambda^2 - \varepsilon}$$

where $\varepsilon = .045 \ \frac{1000\sigma_y}{E}\left(\frac{1 - \mu^2}{.91}\right)^{3/4} \frac{1}{100t/D}$

may be neglected in general to get

$$\psi = \frac{1.30}{\lambda^2}$$

This formula is plotted on figure A-1 and may be compared there with the plot of von Mises' formula and numerous experimental results. One thing which may be seen at this time is the good agreement these formulas have with eachother and the experimental results above a $\lambda$ value of 1.4.

After a decade of relative inactivity, the investigation of stiffened cylindrical pressure vessels was renewed in the early 1950's. The main intent of the investigations of this period was to correct the number of theoretical deficiencies in formulas #92 and #92A. The first two deficiencies were that the formulas predicted linear

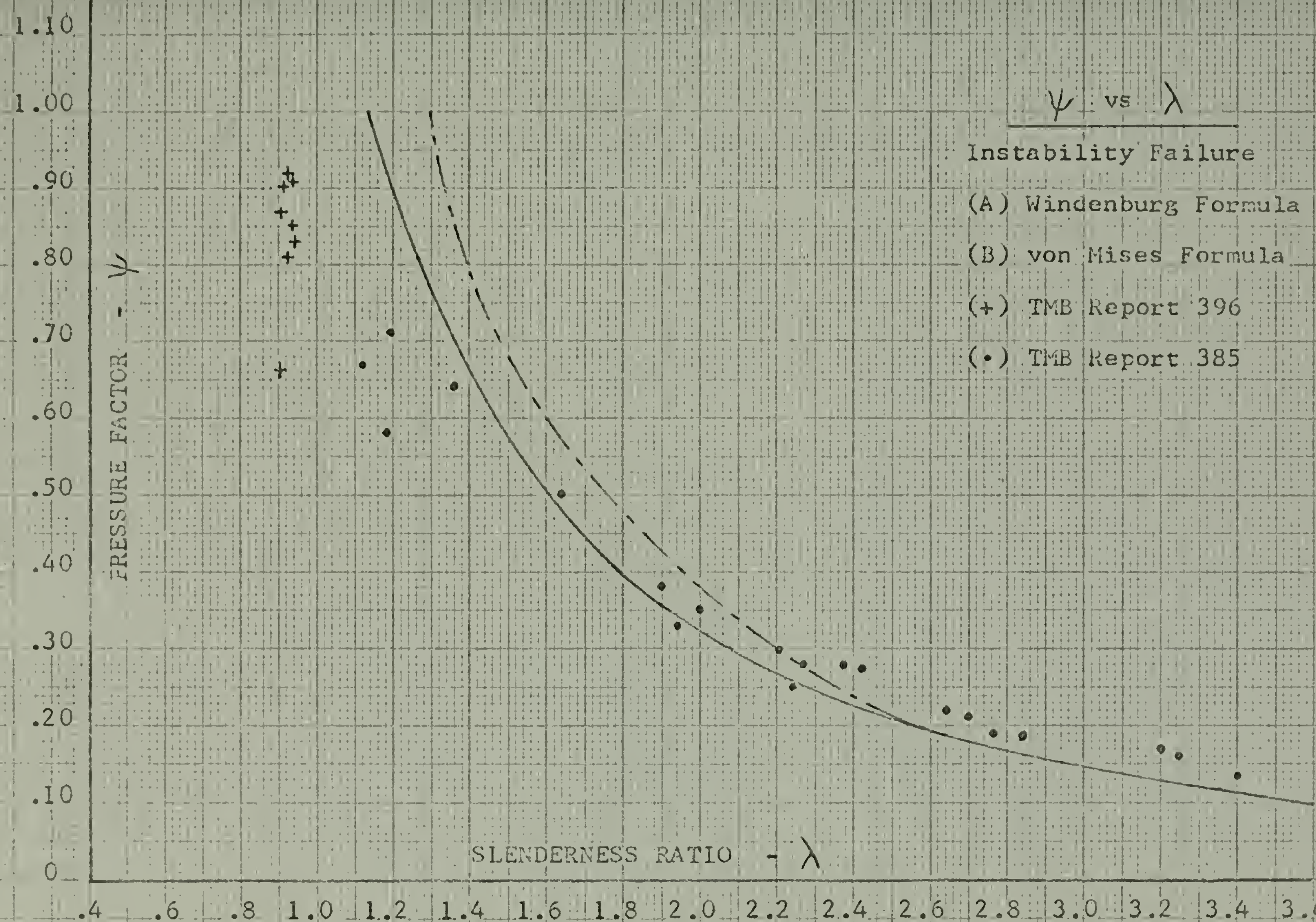

$\psi$ vs $\lambda$
Instability Failure
(A) Windenburg Formula
(B) von Mises Formula
(+) TMB Report 396
(•) TMB Report 385
PRESSURE FACTOR - $\psi$
SLENDERNESS RATIO - $\lambda$
1.10
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6

FIGURE A-1

relationships between pressures and stresses and ignored the effect of moments developed due to pressure at the ends of closed cylinders. Both problems were corrected by Salerno and Pulos (17), who developed stress formulas similar to #92 and #92A but taking both the above moments and the influence of axial stress in the shell tending to expand the frame into account. Their analysis resulted in a non-linear relationship between pressures and stresses which, however, did not alter the results of stress calculations significantly from those found by #92 and #92A.

The next problem found with von Sanden and Gunther's theories had to do with the inadequacy of the maximum principle stress criterion. Use of the Hencky-von Mises yield criterion:

$$\sigma_Y^2 \geq \sigma_T^2 - \sigma_T\sigma_F + \sigma_F^2$$

at the exterior mid-bay point resulted in predictions closer to actual collapse pressure than obtained with the former criterion (9). However, here again collapse pressures were associated with the onset of yielding and no allowance for plastic reserve strength was made.

This condition was corrected by Wenk et. al. (21) in a somewhat haphazard manner by taking the Hencky-von Mises

yield criterion at mid-bay and mid-thickness. Although Wenk termed this a "necessary, if not sufficient empirical yield condition to produce failure", experimental results have since indicated that thought this formula predicts within nine percent of actual collapse pressures, it tends to predict high (9). An interesting observation on the use of the Hencky-von Mises criterion is that for an unstiffened cylindrical shell, $\sigma_T = 2\sigma_F$. This results in failure of such a shell occurring at $\sigma_T = 1.16\sigma_Y$; stated in Windenburg's variables, $\psi = 1.16$.

The use of formulas #92 and #92A in the development of Wenk's formula appears to be the last time these long-standing elastic stress determinators were used in a yield criterion. The elastic stress formulas of Salerno and Pulos (17), having been recognized as more rigorous and slightly more accurate, were used in the next collapse criterion developed by Lunchick. Lunchick utilized the Hencky-von Mises yield criterion, however, he specified that collapse did not occur until the load-carrying capacity of the shell was completely exhausted at the critical mid-bay point. Lunchick's three-hinge mechanism stemmed from his assumption of plastic hinges forming at the frame at the same time a completely plastic hinge had developed at mid-bay. Lunchick's method predicted collapse pressures with better accuracy than any other

in the range of low $\lambda$ values. However, its inclusion of design curves to determine cylinder plastic reserve strength has made it more difficult to apply in computer-based design methods.

It is now generally accepted that stiffened cylindrical shells fail by one of three distinct modes or any combination of them. These modes are (14):

Mode 1. Axisymmetric collapse of the shell between adjacent ring frames.

Mode 2. Asymmetric collapse of the shell between adjacent ring frames (lobar).

Mode 3. Overall asymmetric collapse of the shell and frames together.

Mode one had traditionally been referred to as axisymmetric shell yielding (9,22,23). It has been pointed out, however, that this mode is the result of a combination of yielding and axisymmetric buckling (14,22) and the main difference between this mode and mode two is the difference in resulting collapse patterns, not basic collapse mechanisms. Axisymmetric collapse pressures have been calculated with good accuracy through use of yield criterion alone though, and it is recognized that pre-collapse stresses, as determined by von Sanden and

Gunter (20) or Salerno and Pulos (17), are the primary cause of failure in this mode.

The problem of axisymmetric collapse by elastic buckling has been solved by Salerno and Pulos in (13). Lunchick (10) has investigated axisymmetric failure by inelastic buckling and derived the following expression for it:

$$p_{cr} = \left[\frac{2}{(1-\nu^2)}\left(A_2 - \frac{A_{12}{}^2}{A_1}\right) C^2 E_s \left(\frac{h}{R}\right)^2 + \frac{1}{6(1-\nu^2)} \frac{A_1}{C^2} E_s \left(\frac{h}{R}\right)^2\right] \qquad C = \frac{L^2}{\pi^2 Rh}$$

von Mises' and Windenburg's formulas for the calculation of failure pressure by elastic asymmetric (lobar) buckling between ring frames have been given. A more recent solution to this problem has been found by Reynolds and is outlined in reference (14). Reynolds has also developed a formula for the inelastic buckling of stiffened cylindrical shells which appears to be reasonably accurate, but is consistently high in its calculation of collapse pressures (1).

Mode thr e, collapse by general instability, has been found to be of primary improtance in the preliminary

design of pressure vessels (14). Solution of elastic general instability collapse was first undertaken by Tokugawa, who was also the first to identify this mode. His formula for collapse pressure, (which was later confirmed by Bryant), is as follows:

$$p_{cr} = \frac{Eh}{R_m}\left[\frac{\lambda^4}{(n^2 - 1 + \frac{\lambda^2}{2})(n^2 + d^2)^2}\right](n^2 - 1)\frac{EIe}{R^3 L_f}$$

$$\text{where} \quad \lambda = \frac{\pi R}{L_b}$$

The original Tokugawa formula as given in (19) has a number of extra terms in it which are seen to be negligible for geometries of general interest.

The lengthy formulas developed by Kendrick for elastic general instability are given in (14), however, a graphical solution of Kendrick's equation as developed by Reynolds, and refined by Ball, is included as figures A-2 and A-3. In this solution, the sum of two effects, each represented by a graph, is used to determine collapse pressure. Figure 2 is the shell effect and figure 3 the frame effect which reflects the bending stiffness of the cylinder. Figures 2 and 3 also indicate the Bryant-Tokugawa solution. Collapse pressure by either set of curves is found in the following manner:

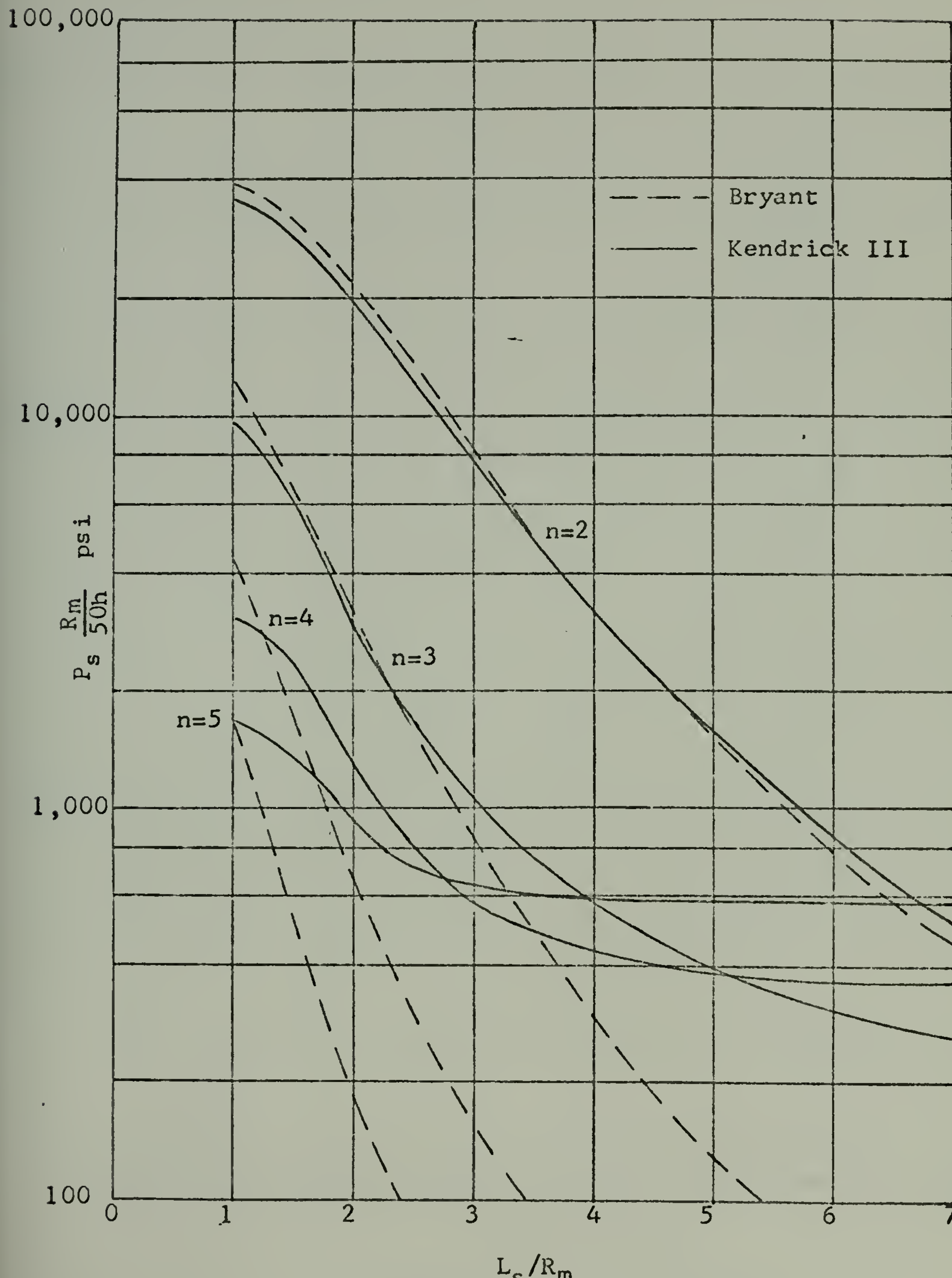



FIGURE A-2 (SHELL EFFECT)

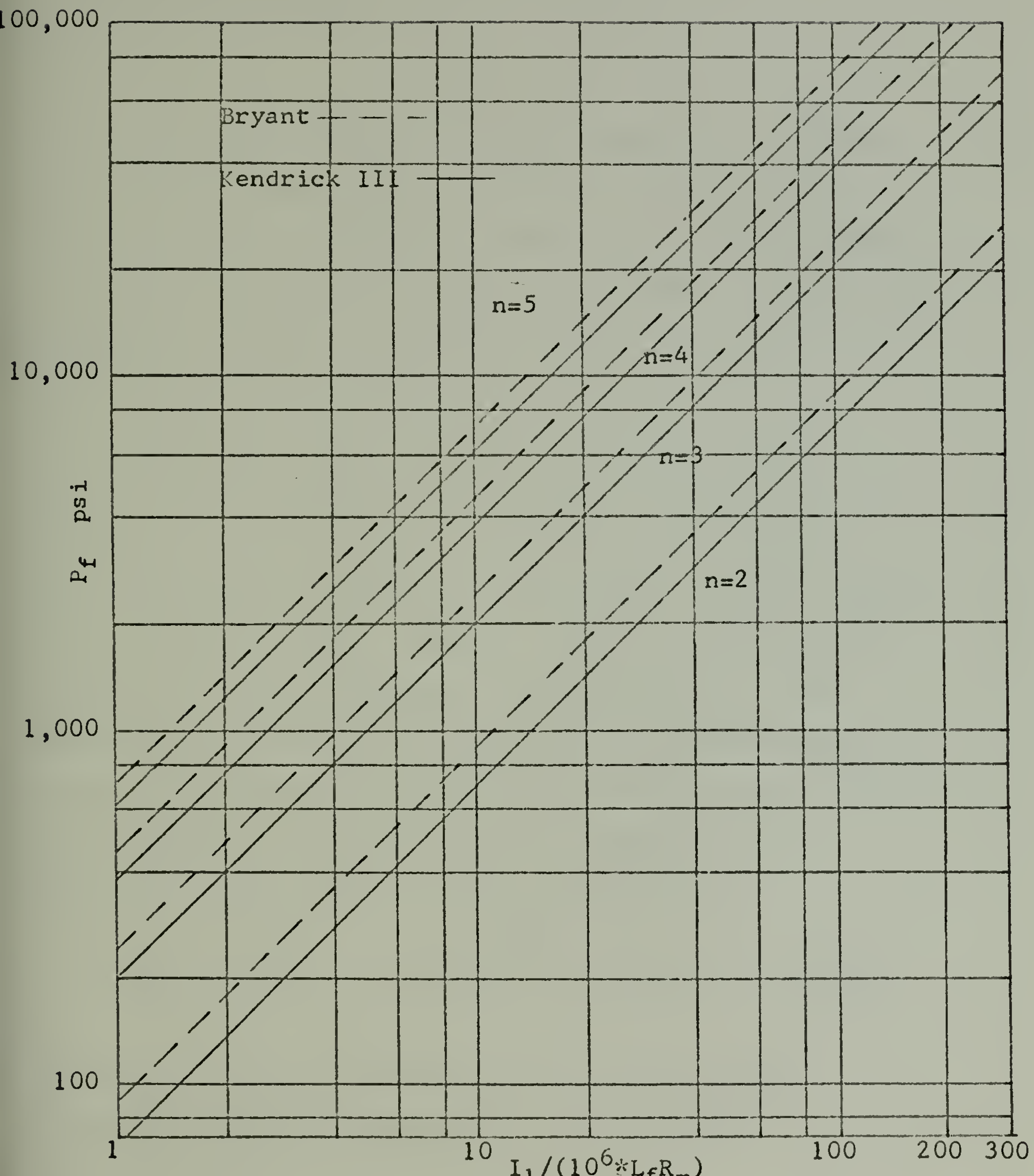



FIGURE A-3 (FRAME EFFECT)

$$p_c = p_s\left(\frac{L_b}{R}, \frac{h}{R}, n\right) + p_f\left(\frac{Ie}{R^3 L_f}, n\right)$$

In order to round out this development of stiffened cylindrical pressure vessel collapse theories, mention must be made of the effect of actual physical manufacturing conditions. In all but the smallest and most painstakingly fabricated cylinders, imperfect production processes are the cause of a number of collapse strength modifiers. Those modifiers introduced into the material itself are in the form of stresses due to fabrication, rolling, and welding of flat plating. In thick homogeneous plating, the homogeneity itself may be questionable and thus cause stress concentrations. A major strength modifier is the imperfect circularity of most welded cylinders. This modifier has received some theoretical consideration as it has a considerable effect on the resistance of a stiffened cylinder to failure by general instability. A formula for this is due in part to Kendrick (2):

$$\sigma_{max} = \frac{p_1 R}{A_b} b \left[\frac{1 + \frac{.85}{B}}{1 + \beta}\right] + \frac{Ec}{R_1{}^2} (n^2-1) e \left(\frac{p_1}{p_{cr} - p_1}\right)$$

As long as the $\sigma_{max}$ is below material yield stress, collapse by general instability is not indicated.

This section has been included to give a brief background of the development of theoretical collapse predictors for stiffened cylindrical shells. In particular, a history of the variables of $\lambda$ and $\psi$ is helpful in understanding their use in this paper.

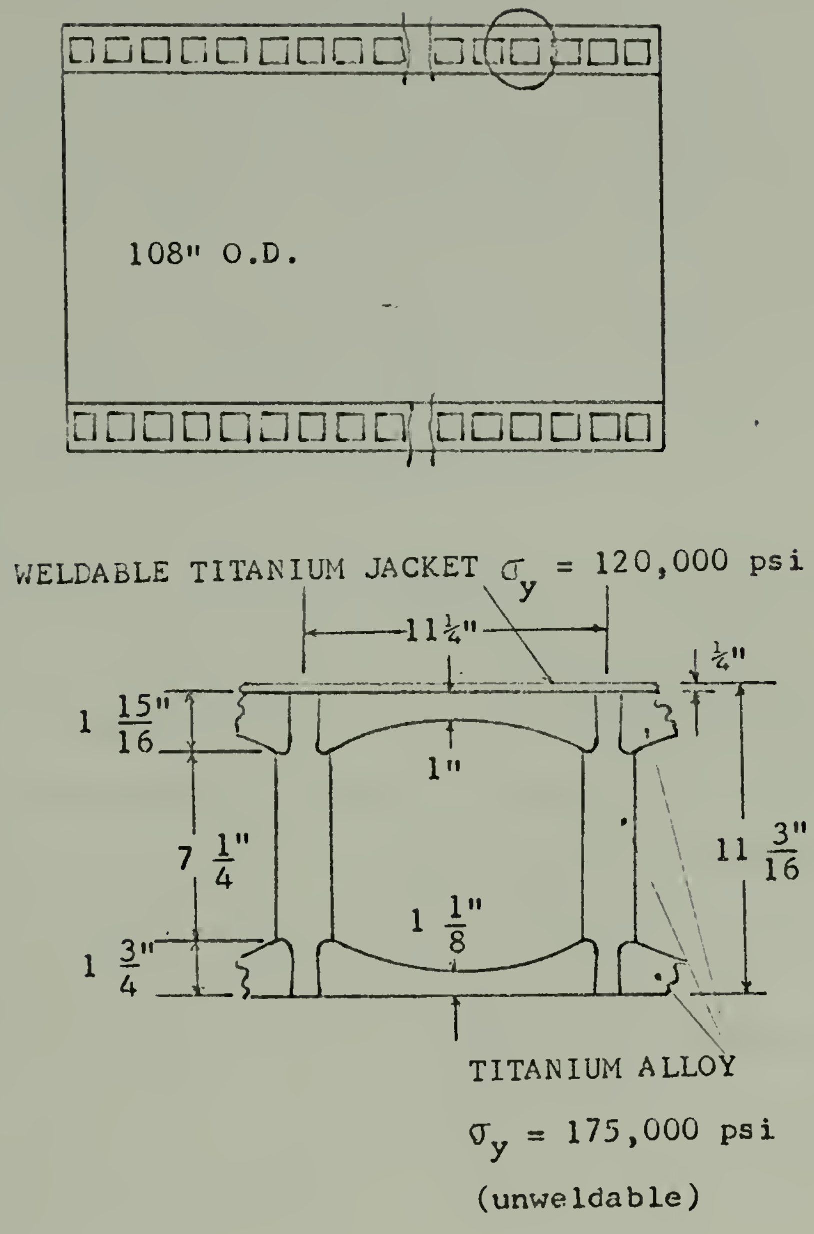
# APPENDIX B

## DISCUSSION OF VERY LOW LAMBDA DESIGN

As noted in Section IV, overall structural efficiency approaches its highest value as $\psi$ goes to 1.0 and $\lambda$ goes to 0. For T-shaped frames of the geometry used in this report, $\lambda$ values in the area below .2 may be considered very low lambda design. For other frame geometries, this may be higher. For example, the sandwich construction in figure A-4 has a $\lambda$ value of about .337. The frames used there have thicker webs than flanges, whereas the reverse is true for the frames of this report. The membrane shell design, and other complications resulting from the use of high strength titanium also plays a part in deciding the optimum $\lambda$ value.

It was noted in Section IV that the danger of general instability collapse would be greatly increased by the use of a very low lambda design. This has been shown to be true in model tests of cylindrical shells with geometries similar to that of figure A-4. In fact, in the testing of these models, a new mode of collapse was observed - that of plastic general instability - and new theories had to be developed to predict it. It may also be observed that collapse of these models took place at $\psi$ value in excess of 3.0. This suggests that at very low lambda

values, $\psi$ , becomes a rather meaningless term unless the framing is very small in comparison to the shell thickness. Indeed, the large effect of framing at these $\lambda$ values throws into doubt the meaningfulness of $\lambda$ itself. This doubt has been substantiated but not fully explained in reference (6).




SCHEMATIC SKETCH of an OCEANOGRAPHIC VEHICLE WITH . 9-FOOT OUTER DIAMETER from DTMB 1677

FIGURE A-4

# APPENDIX C

## FRAME PARAMETERS

A T-shaped frame was used in the calculations leading to figures 5-10. The proportions of this frame were taken from a work by McGinley (11) and are as follows:

| | | |
|---|---|---|
| frame width/frame depth | = | 1.0 |
| frame width/web thickness | = | 8.5 |
| flange thickness/web thickness | = | 1.7 |

Figure A-5 is a graph of b/D versus $\lambda$ for the frames for both formulas #92A and Wenk's. Since the geometry of optimized vessels was found to be a function of required collapse pressure,and not required diameter, the optimized frame dimensions should be proportional to the major cylinder dimensions, hence the b/D parameter. Figure A-4 may be used in conjunction with figures 8, 9, and 10 to determine the frame dimensions. Care should be taken, however, to match yield formulas and frame factors when using these figures together.

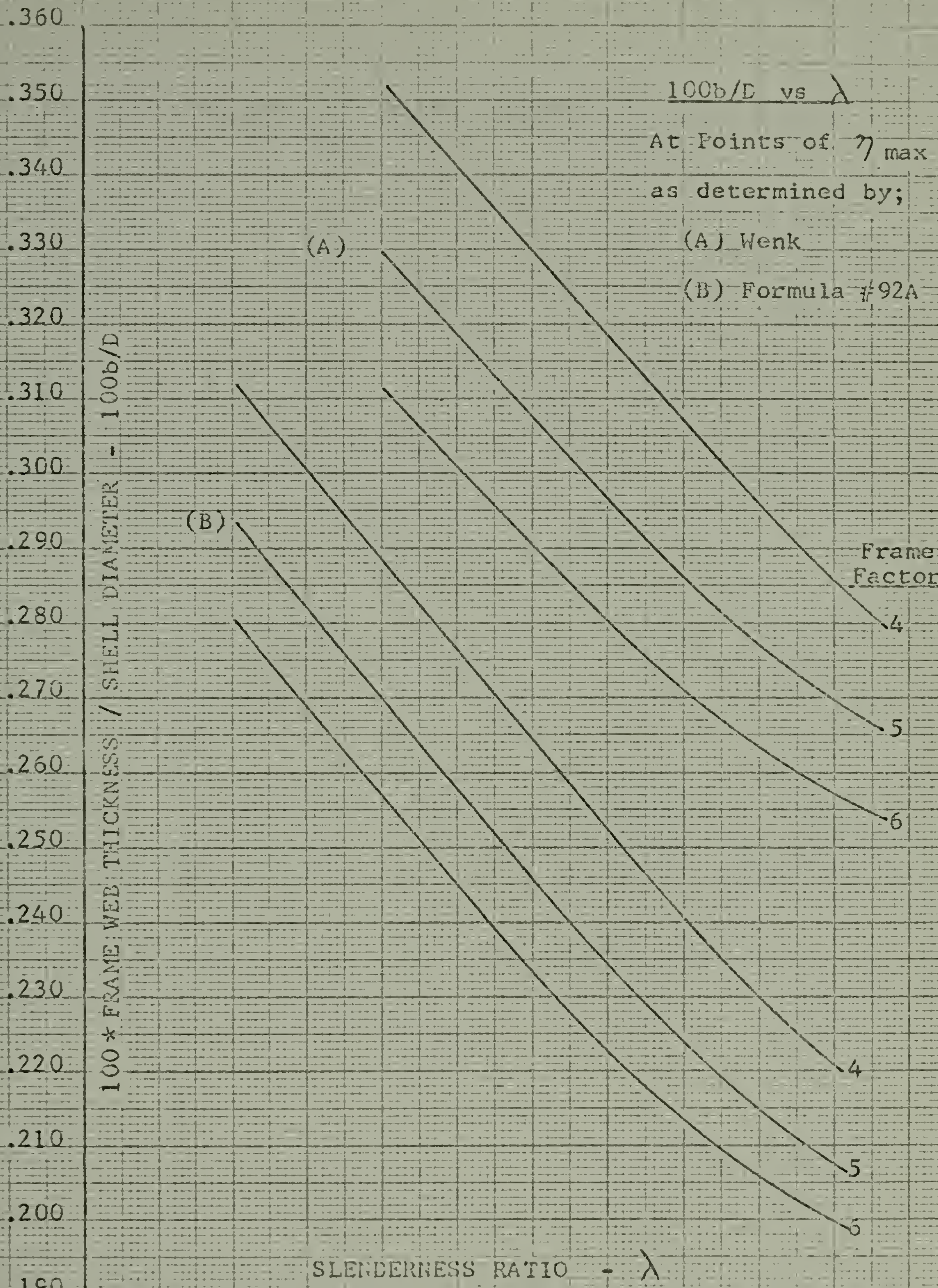

100b/D vs $\lambda$
At Points of $\eta_{max}$
as determined by;
(A) Wenk
(B) Formula #92A
(A)
(B)
Frame Factor
4
5
6
4
5
6
100 * FRAME WEB THICKNESS / SHELL DIAMETER - 100b/D
SLENDERNESS RATIO - $\lambda$
.360
.350
.340
.330
.320
.310
.300
.290
.280
.270
.260
.250
.240
.230
.220
.210
.200
.190
.3
.4
.5
.6
.7
.8

FIGURE A-5

# APPENDIX D

## EXAMPLES OF DESIGN ITERATION

Example 2:

This example is included to indicate the results of the iterative method when applied to relatively low pressure vessels.

Given: Use of #92A Frame Factor 5

$\sigma_y$ = 30,000 psi

$p_c$ = 400 psi

D = 40 ft.

Steps:

1. Initial $h/D_m$ guess from hoop stress

$(h/D_m)$ = 400/(2*30,000)

$(h/D_m)$ = .00666

$\lambda$ = .495

$\psi$ = 1.30

$p_c$ = 1.30*400 = 520 psi
which is too high

2. Reduce $h/D_m$ by division

$p_{c1} = \dfrac{400}{1.3} = 308$ psi

$(h/D_m)$ = .00512

$\lambda$ = .565

$\psi$ = 1.39

$p_c$ = 1.39*308 = 428 psi
which is still too high

$\psi_{act} = \dfrac{428}{400} = 1.07$

3. Reduce $h/D_m$ again

$$p_{c2} = \frac{308}{1.07} = 288 \text{ psi}$$

$$(h/D_m) = .00480$$

$$\lambda = .584$$

$$\psi = 1.41$$

$$p_c = 1.41 * 288 = 406 \text{ psi}$$

which is close enough

Answer:

$$\lambda = .584$$

$$h/D_m = .00480$$

$$\eta = 1.27 * \frac{30{,}000}{80{,}000}$$

$$\eta = .476$$

$$\psi = 1.41$$

Results:

$$D = 40 \text{ ft.}$$

$$h = 2.3 \text{ in.}$$

$$L = 4.5 \text{ ft.}$$

$$\frac{Wt}{\Delta} = \frac{p_c}{3720\eta} = \frac{406}{3720(.476)}$$

$$\frac{Wt}{\Delta} = .23$$

Example 3:

This example is exactly like example 2 except that a higher strength material is used.

Given: Use of #92A Frame Factor 5

$\sigma_y$ = 50,000 psi

$p_c$ = 400 psi

D = 40 ft.

Answer: $\lambda$ = .825

$h/D_m$ = .00232

$$\eta = 1.11 * \frac{50,000}{80,000}$$

$\eta$ = .694

$\psi$ = 1.735

Results: D = 40 ft.

h = 1.11 in.

L. = 1.83 ft.

$\frac{Wt}{\Delta}$ = .157

Example 4:

This example is included to indicate the use of the frame parameter curves in Appendix C.

Given: Use of #92A Frame Factor 6

$\sigma_y$ = 100,000 psi

D = 40 ft.

$p_c$ = 1500 psi

Steps:

1. Initial $h/D_m$ guess

$(h/D_m)$ = 1500/(2*100,000)

$(h/D_m)$ = .007

$\lambda$ = .48

$\psi$ = 1.33

$p_c$ = 1.33*1500 = 2000 psi

2. Reduce $h/D_m$

$p_{cl} = \frac{1500}{1.33} = 1130$

$(h/D_m)$ = 1130/(2*100,000)

$(h/D_m)$ = .0056

$\lambda$ = .54

$\psi$ = 1.40

$p_c$ = 1.40*1130 = 1580 psi

$\psi_{act} = \frac{1580}{1500} = 1.05$

3. Reduce $h/D_m$ again

$p_{c2} = \frac{1130}{1.05} = 1075$ psi

$(h/D_m) = 1075/(2*100{,}000)$

$(h/D_m) = .00537$

$\lambda = .55$

$\psi = 1.41$

$p_c = 1.41*1075 = 1515$ psi

close enough

Answer:

$\lambda = .55$

$(h/D_m) = .00537$

$\eta = \frac{100{,}000}{80{,}000} = 1.59$

$\psi = 1.41$

$\frac{b}{D} = .00248$

Results:

$D = 40$ ft.

$h = 2.6$ in.

$L = 1.42$ ft.

$\frac{Wt}{\Delta} = .252$

Frame Dimensions:

$b = 1.18$ in.

$w = 8.05$ in.

$f = 10.1$ in.

$t_{\frac{1}{2}} = 2.05$ in.

# APPENDIX E

## FORMULAS AND DERIVATIONS

In this appendix, the formulas used in the computer programs in Appendix F are listed. Also, the derivation of formulas (A) and (B), Section II, and the solution of the Hencky-von Mises criterion for unstiffened tubes are presented.

FRAME FORMULAS: The formulas below are used to predict frame failure by instability. Collapse of the frame by yielding was not investigated as the frames were not required to hold up after shell failure and the maximum stress in the frame is always less than the maximum stress in the shell (19). However, this stress relationship may not hold in the very low lambda region.

Foppl Formula:

$$p_c = \frac{24EI_b}{D_b{}^3L_1}$$

Formula #88
von Sanden and Gunther:

$$p_c = \frac{24EI_b\ (1+\beta)}{D_b{}^3b\ \left(1 + \frac{.85\beta}{B}\right)}$$

$$\text{where } B = \frac{bh}{A + bh}$$

$$\theta = 1.82 \frac{L/D_m}{h/D_m}$$

$$\beta = 2\frac{L}{b}\frac{NB}{\theta}$$

$$N = \frac{\cosh\theta - \sin\theta}{\sinh\theta + \sin\theta}$$

SHELL FORMULAS: The first formula below predicts collapse of the shell between frames due to instability. The next four predict collapse due to yielding of the material according to different yield criteria.

Formula #92
von Sanden and Gunther

Failure due to axial stress exceeding yield stress:

$$p_c = 2\frac{h\sigma}{D_m}y \left( \frac{1}{.5 + 1.81KC} \right)$$

$$K = \frac{\sinh\theta - \sin\theta}{\sinh\theta + \sin\theta}$$

$$C = \frac{.85 - B}{1 + \beta}$$

B and $\beta$ are defined in Formula #88

Formula #92A
von Sanden and Gunther

Failure due to circumferential stress exceeding yield stress:

$$p_c = 2\frac{h}{D_m}\sigma_y \left( \frac{1}{1 + HC} \right)$$

$$H = -2 \frac{(1 + M) \sinh\frac{\theta}{2} \cos\frac{\theta}{2} + (1-M) \cosh\frac{\theta}{2} \sin\frac{\theta}{2}}{\sinh\theta + \sin\theta}$$

$$M = \frac{3\mu^2}{1 - \mu^2}$$

C is defined in Formula #92

Wenk's Formula

This formula was actually developed by Wenk and others at DTMB using #92 and #92A to determine the principle stresses. These were applied at mid-bay and mid-thickness using the Hencky-von Mises yield criterion.

$$p_c = 2\frac{h}{D_m}\sigma_Y \frac{1}{\sqrt{F^2 - .5F + .25}}$$

$$F = 1 + 2QC$$

$$Q = - \frac{\sinh\frac{\theta}{2} \cos\frac{\theta}{2} + \cosh\frac{\theta}{2} \sin\frac{\theta}{2}}{\sinh\theta + \sin\theta}$$

$\theta$ and C are defined in Formula #92

Lunchick's Formula

This formula uses the principle stress relations developed by Salerno and Pulos rather than #92 and #92A (9). It applies them to the mid-bay location using the Hencky-von Mises criterion, however, failure is not predicted until the yielding which occurs first in the outer surface develops into a fully plastic hinge. Lunchick's formula predicts this yield pressure, but curves (included here

as figure A-6) must be used to determine the actual collapse pressure.

$$p_c = 2\frac{h}{D_m}\sigma_y \frac{1}{\sqrt{X^2 - XZ + Z^2}}$$

$$X = 1 + HC$$

$$Z = .5 + 3.62TC$$

$$T \cong - \frac{\sinh\frac{\theta}{2}\cos\frac{\theta}{2} - \cosh\frac{\theta}{2}\sin\frac{\theta}{2}}{\sinh\theta + \sin\theta}$$

$\theta$, C, and H are defined in Formula #92A

Entry into the curves requires computation of the following ratios:

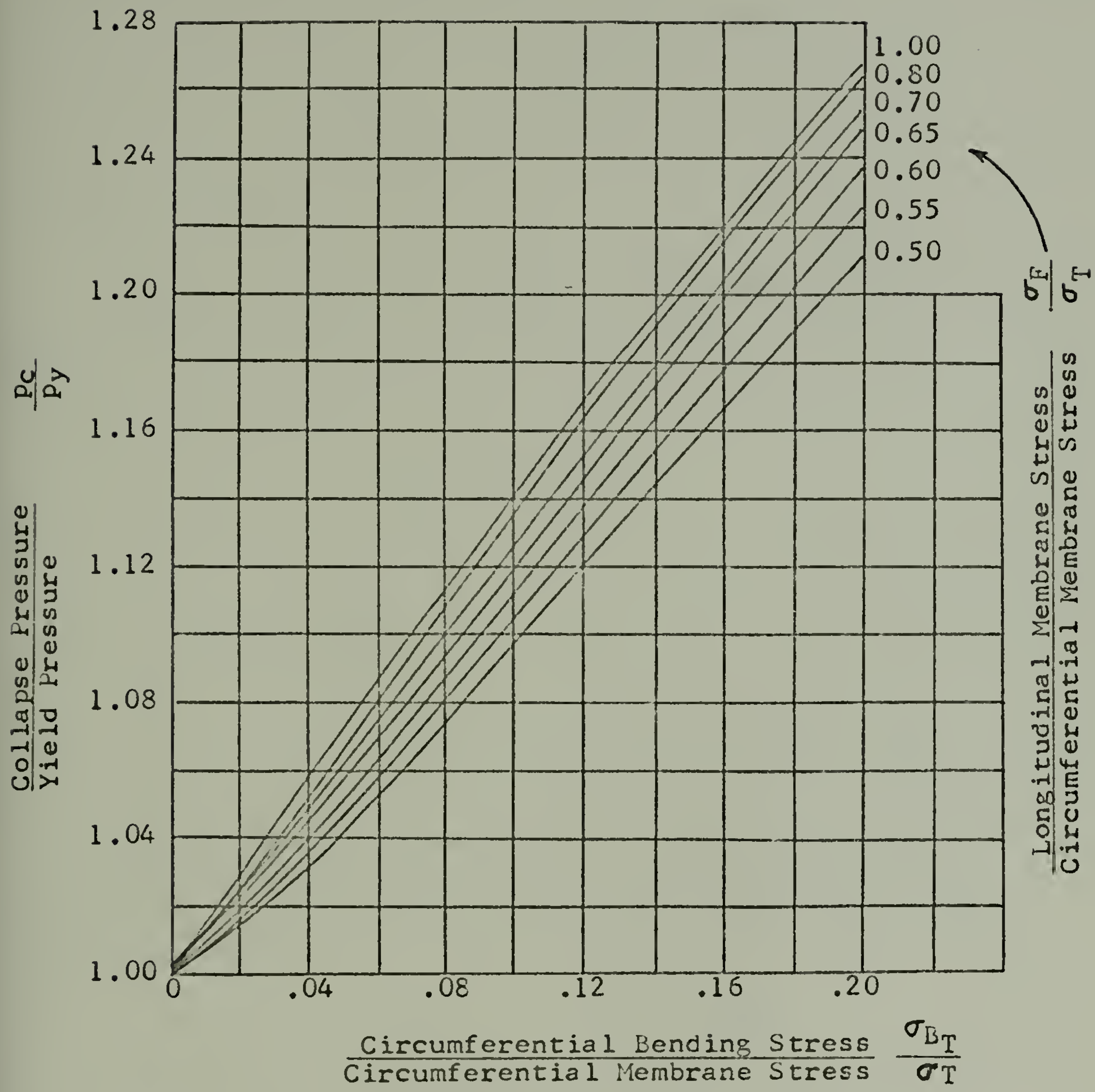
$$\frac{\text{Circumferential Bending Stress}}{\text{Circumferential Membrane Stress}} = \frac{\sigma_{BT}}{\sigma_T}$$

$$\frac{\text{Longitudinal Membrane Stress}}{\text{Circumferential Membrane Stress}} = \frac{\sigma_F}{\sigma_T}$$

$$\sigma_F = \frac{D_m\, p_c}{4h}$$

$$\sigma_T = 2\sigma_F(1 + 2QC)$$

$$\sigma_{BT} = 2\sigma_F\, TC$$




Curves for Lunchick's Collapse Pressure
Factor - From DTMB Report 1291

FIGURE A-6

GENERAL INSTABILITY FORMULAS: While Tokugawa's formula and graphs of Kendrick's third solution for general instability have been given in Appendix A, Bryant's simplification of Kendrick's first solution has been used in the computer program.

Bryant's Formula:

$$p_{cr} = 2\frac{h}{D_m}E\left(\frac{\lambda^4}{(n^2 - 1 + \frac{\lambda^2}{2})(n^2 + \lambda^2)}\right)$$

$$+ \frac{n^2 - 1}{R_m{}^3 L_1} EI_1$$

$$\lambda = \frac{\pi R}{L_b} \quad \text{an integer}$$

$n$ = the integer which gives the lowest $p_{cr}$.

Since out of roundness effects general instability collapse, a formula taken from (2) has been used in the following form:

$$\sigma_{max} = \frac{p_c R}{A_b} b \left[\frac{1 + \frac{.85\beta}{B}}{1 + \beta}\right]$$

$$+ \frac{Ec}{R_1{}^2}(n^2 - 1)e\left(\frac{p_c}{p_{cr} - p_c}\right)$$

$e$ = eccentricity

$p_c$= desired collapse pressure

$n$, $p_{cr}$, $\beta$, and $B$ are determined in previous formulas.

$\lambda_m$ FROM PRINCIPLE STRESS CRITERION AT MID-BAY EXTERIOR, FORMULA #92A

1. Formula #92A

$$p_c = \frac{2\frac{h}{D_m}\sigma y}{1 + H\left(\frac{.85 - B}{1 + \beta}\right)}$$

2. $$\psi = \frac{p_c}{2\frac{h}{D_m}\sigma y}$$

$$\psi = 1 \text{ @ } \frac{1}{1 + H\left(\frac{.85 - B}{1 + \beta}\right)} \doteq 1$$

$$1 + H\left(\frac{.85 - B}{1 + \beta}\right) = 1$$

$$H\left(\frac{.85 - B}{1 + \beta}\right) = 0$$

$$H = 0$$

$$H \cong - \frac{3 \sinh\frac{\theta}{2} \cos\frac{\theta}{2} + \cosh\frac{\theta}{2} \sin\frac{\theta}{2}}{\sinh + \sin}$$

$$H = 0 \text{ @ } \theta = 3.74 \text{ radians}$$

$$\theta = (1.82\ L/D_m)/\sqrt{h/D_m}$$

$$L/D_m = 2.055\sqrt{h/D_m}$$

$$h/D_m = .236(L/D_m)^2$$

3. $$\lambda = \sqrt[4]{\frac{(L/D_m)^2}{(h/D_m)^3}} \sqrt{\frac{\sigma_y}{E}}$$

$$\lambda_m = \sqrt{\frac{2.055\ D_m}{h} \frac{\sigma_y}{E}}$$

Equation (A), Section II

$\lambda_m$ FROM HENCKY-VON MISES CRITERION AT MID-BAY AND MID-THICKNESS,

VON SANDEN AND GUNTHER FORMULAS

1. Wenk's Formula

$$p_c = \frac{2h\sigma_y}{D_m} \quad \frac{1}{\sqrt{F^2 - \frac{F}{2} + \frac{1}{4}}}$$

$$F = 1 + 2Q \left(\frac{.85 - B}{1 + \beta}\right)$$

$$Q = - \frac{\sinh\frac{\theta}{2}\cos\frac{\theta}{2} + \cosh\frac{\theta}{2}\sin\frac{\theta}{2}}{\sinh\theta + \sin\theta}$$

2. $$\psi = \frac{p_c}{2\frac{h}{D_m}\sigma_y}$$

$$\psi = 1.154 @ \frac{1}{\sqrt{F^2 - \frac{F}{2} + \frac{1}{4}}} = 1.154$$

$$F^2 - \frac{F}{2} + \frac{1}{4} = .75$$

$$F = .25 \pm .75$$

$$\therefore 1 = 1 + 2Q \left(\frac{.85 - B}{1 + \beta}\right)$$

$$\therefore Q = 0 @ \theta = 4.68 \text{ radians}$$

$$\theta = (1.82\ L/D_m) \big/ \sqrt{h/D_m}$$

$$L/D_m = 2.565 \sqrt{h/D_m}$$

$$h/D_m = .152\ (L/D_m)^2$$

3. $$\lambda = \sqrt[4]{\frac{(L/D_m)^2}{(h/D_m)^3}} \sqrt{\frac{\sigma_y}{E}}$$

$$\lambda_m = \sqrt{\frac{2.565\ D_m}{h} \quad \frac{\sigma_y}{E}}$$

Equation (B), Section II

SHELLS [illegible] WIDELY SPACED STIFFENERS

1. He[illegible]-von Mises Criterion

$\sigma_1$, [illegible], $\sigma_3$ - three principle stresses

$$2\sigma_y = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

2. For [illegible] unstiffened tube:

$$\sigma_1 = 2\sigma_2 \qquad \sigma_3 = 0$$

$$2\sigma_y^2 = \sigma_2^2 + \sigma_2^2 + 4\sigma_2^2$$

$$\sigma_y^2 = 3\sigma_2^2$$

$$\sigma_2 = \frac{1}{1.732\ \sigma_y} = .577\sigma_y$$

$$\sigma_1 = \frac{2}{1.732\ \sigma_y} = 1.154\sigma_y$$

# APPENDIX F

## COMPUTER PROGRAMS

Two computer programs were used to produce theoretical data for this report. Program 1 utilizes equation (A) of Section II to determine curves of $h/D_m$ versus $\lambda_m$ for eight values of $\sigma_y$. Since equation (A) is based on the $\theta$ value at which the variable H in formula #92A goes to zero, the program first solves for that $\theta$ and then goes on to vary $h/D_m$ and $\sigma_y$, solving for $\lambda_m$. This program was used only to define the upper limits of the yield failure area and so, is not actually necessary for the iterative optimization proceedure of Section IV. A flow chart and variable chart are included with the program listing and printout.

Program 2 follows a scatter-gun philosophy in that it determines collapse pressures of given geometries for all three modes of failure by various formulas. All of the formulas are mentioned in Appendix A and completely defined in Appendix E. The input of this program consists of outside diameter, desired range of collapse pressures, frame proportions, material properties, and range of values. Shell thickness is determined by the hoop stress relation and frame size by a modified Foppl formula. Program 2 was not intended to be used to produce a complete

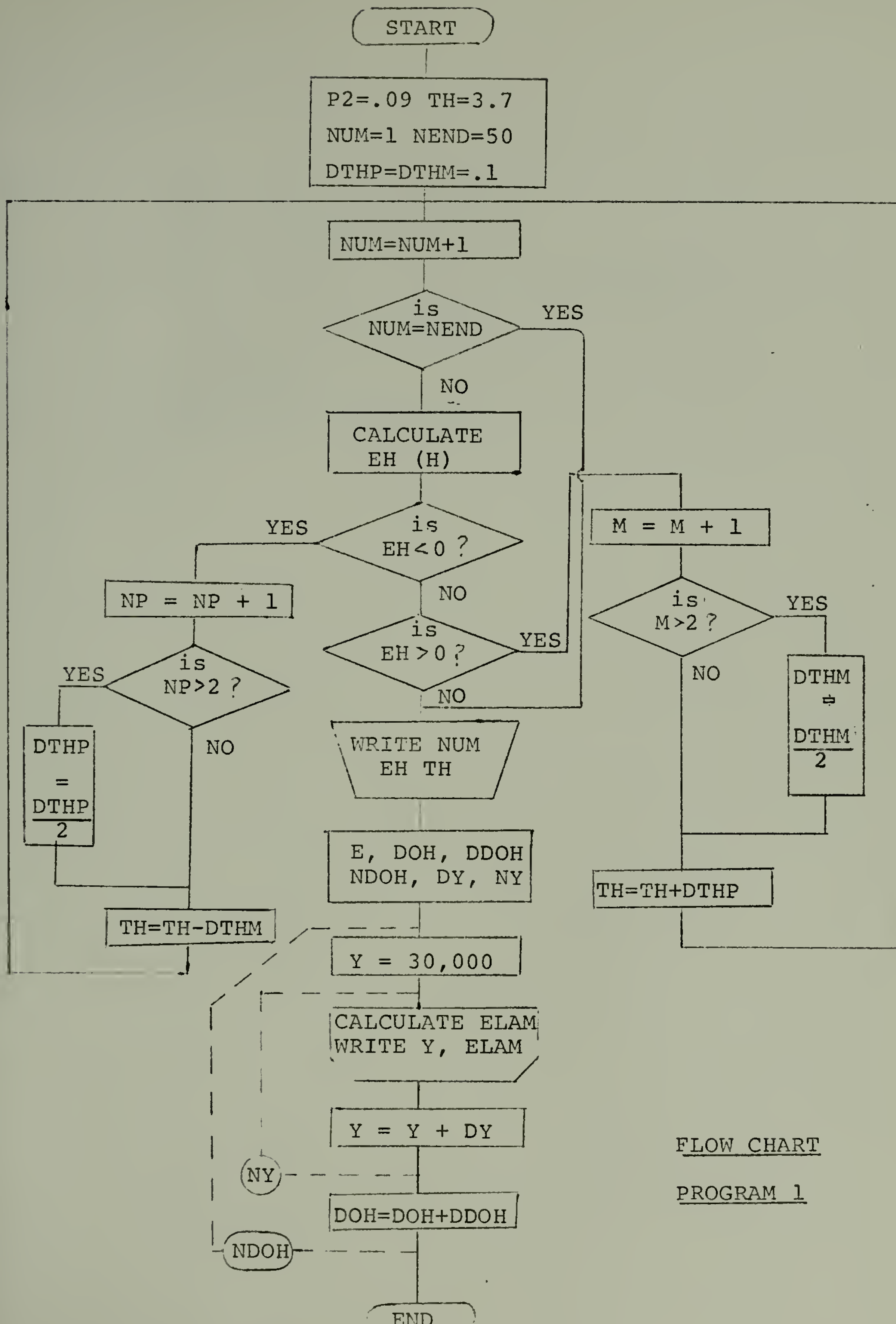
optimized design for a given set of requirements. Its main use, in solving for and printing collapse pressures for all modes, is pointing up trends and areas where various formulas fail. It has provisions for any number of solutions to be printed and any geometry to be investigated. In short, it is meant to be used as a tool by a designer who optimizes, using judgment and experience as well as computer time. A flow chart and variable chart are included with the program listing and a sample of its printout.

PROGRAM 1

```
C     THIS PROGRAM  DETERMINES LAMBDA MAX FOR CHANGING H/D
C     AND YIELD STRESS USING A FORMULA DERIVED FROM 92A.
C
C
      P2 = .09
      NUM = 1
      NEND = 50
      TH = 3.7
      DTHP = .1
      DTHM = .1
    1 NUM = NUM + 1
      IF(NUM.EQ.NEND) GO TO 30
      THT = TH/2.
      DEN = SINH(TH)+SIN(TH)
      EH0 = (1.+(3*P2/(1-P2))**.5)*SINH(THT)*COS(THT).
      EH1 = (1.-(3*P2/(1-P2))**.5)*COSH(THT)*SIN(THT)
      EH = -2*(EH0+EH1)/DEN
      IF(EH) 19,30,9
    9 NP = NP+1
      IF(NP.EQ.3) GO TO 10
      DTHP = DTHP/2
   10 TH = TH-DTHM
      M = 1
      GO TO 1
   19 M = M+1
      IF(M.EQ.3) GO TO 20
      DTHM = DTHM/2
   20 TH = TH + DTHP
      NP = 1
      GO  TO 1
   30 WRITE(6,180) NUM,EH,TH
  180 FORMAT(' AFTER',I3,' ITERATIONS, H IS',F8.6,' AND THETA
      E = 30000000.0                                          IS',F6.3)
      DOH = 0.0
      DDOH = 40.0
      NDOH = 15
      DY = 10000.0
      NY = 8
      DO 50 N = 1,NDOH
      WRITE(6,205) DOH
      Y = 30000.0
      DO 40 M = 1,NY
      ELAM = (TH/1.82*DOH*Y/E)**.5
      WRITE(6,210) Y,ELAM
   40 Y = Y+DY
   50 DOH = DOH + DDOH
  205 FORMAT(' LAMBDAS FOR DM/H = ',F7.2,' ARE ')
  210 FORMAT(2F12.4)
      END
```

PRINTOUT OF PROGRAM 1 (Sample)

```
AFTER 50 ITERATIONS, H IS-.000000 AND THETA IS 3.74
LAMBDAS FOR DM/H =      0.0  ARE
 30000.0000          0.0
 40000.0000          0.0
 50000.0000          0.0
 60000.0000          0.0
 70000.0000          0.0
 80000.0000          0.0
 90000.0000          0.0
100000.0000          0.0
LAMBDAS FOR DM/H =    40.00 ARE
 30000.0000          0.2867
 40000.0000          0.3311
 50000.0000          0.3702
 60000.0000          0.4055
 70000.0000          0.4380
 80000.0000          0.4683
 90000.0000          0.4967
100000.0000          0.5235
LAMBDAS FOR DM/H =    80.00 ARE
 30000.0000          0.4055
 40000.0000          0.4683
 50000.0000          0.5235
 60000.0000          0.5735
 70000.0000          0.6194
 80000.0000          0.6622
 90000.0000          0.7024
100000.0000          0.7404
LAMBDAS FOR DM/H =   120.00 ARE
 30000.0000          0.4967
 40000.0000          0.5735
 50000.0000          0.6412
 60000.0000          0.7024
 70000.0000          0.7587
 80000.0000          0.8110
 90000.0000          0.8602
100000.0000          0.9068
LAMBDAS FOR DM/H =   160.00 ARE
 30000.0000          0.5735
 40000.0000          0.6622
 50000.0000          0.7404
 60000.0000          0.8110
 70000.0000          0.8760
 80000.0000          0.9365
 90000.0000          0.9933
100000.0000          1.0470
```




FLOW CHART

PROGRAM 1

NOMENCLATURE OF PROGRAM 1

| | | |
|---|---|---|
| DOH | - | $D_m/h$ |
| DDOH | - | Incremental change in $D_m/h$ |
| NDOH | - | Number of changes in $D_m/h$ |
| E | - | Young's Modulus |
| EH | - | Variable H of yield failure formulas |
| ELAM | - | $\lambda_m$ |
| P2 | - | Poisson's Ratio squared |
| TH | - | Variable $\theta$ of yield failure formulas |
| Y | - | Material $\sigma_y$ |
| DY | - | Incremental change in $\sigma_y$ |
| NY | - | Number of changes in $\sigma_y$ |

PROGRAM 2

```
C     THIS PROGRAM DETERMINES THEORETICAL COLLAPSE PRESSURES
C     FOR STIFFENED CYLINDRICAL PRESSURE VESSELS. THREE MODES
C     OF COLLAPSE ARE INVESTIGATED WITH EMPHASIS ON YIELD
C     PRECIPITATED BUCKLING BETWEEN THE FRAMES. FRAME SIZE
C     IS DETERMINED USING A MODIFIED FOPPL FORMULA. ALL INPUTS
C     ARE HANDLED BY MAIN PROGRAM CARDS - NO DATA CARDS ARE
C     USED.

      MF = 6
      D = 480.0
      PC = 300.0
      DO 700 NU = 1,14
      Y = 80000.0
      E = 30000000.0
      R1 = 1.0
      R2 = .11764
      R3 = 1.7
      P = .3
      P2 = P**2
      SR = .38
      IF(PC.LT.1000.0) SR = .46
      DSR = .02
      NSR = 20
      T = D*PC/(2*Y)
      DM = D-T
      TOD = T/DM
      WRITE(6,160) T,TOD
      DO 500 IN = 1,NSR
      EL = SR**2*DM*E/Y*(T/DM)**1.5
      FRAMD1 = 12.0
      DFRP = 1.0
      DFRM = 1.0
      NUM = 1
      NEND = 70
      RATIO = EL*PC/(24*E*MF)
    1 NUM = NUM + 1
      IF(NUM.EQ.NEND) GO TO 30
      F = FRAMD1*R1
      B = FRAMD1*R2
      T2 = B*R3
      W = FRAMD1 - T2
      YB = YENTRD(B,T,B,W,T2,F)
      YBI = YMOM(B,T,B,W,T2,F,YB)
      DBT = (D-2*(YB+T))**3
      RAT = YBI/DBT
      IF(RAT-RATIO) 19,30,9
    9 NP = NP+1
```

```
      IF(NP.EQ.3) GO TO 10
      DFRP = DFRP/2
   10 FRAMD1 = FRAMD1 - DFRM
      M = 1
      GO TO 1
   19 M = M+1
      IF(M.EQ.3) GO TO 20
      DFRM = DFRM/2
   20 FRAMD1 = FRAMD1 + DFRP
      NP = 1
      GO TO 1
   30 YF = YENTRD(0.0,0.0,B,W,T2,F)
      Y1 = YENTRD(EL,T,B,W,T2,F)
      YI = YMOM(0.0,0.0,B,W,T2,F,YF)
      Y1I = YMOM(EL,T,B,W,T2,F,Y1)
      YBIOYI = YBI/YI
      Y1IOYI = Y1I/YI
      RONE = (D-2*(T+Y1))/2
      DB = D-2*(T+YB)
      A = F*T2 + B*W
      AB = A+B*T
      EB = B*T/AB
      THETA = 10*(12-12*P2)**.25
      TH = THETA*EL/DM/(100*T/DM)**.5
      THT = TH/2
      DEN = SINH(TH)+SIN(TH)
      EN = (COSH(TH)-COS(TH))/DEN
      EK = (SINH(TH)-SIN(TH))/DEN
      BETA = 2.*EL*EN*EB/(B*TH)
      EH0 = (1+(3*P2/(1-P2))**.5)*SINH(THT)*COS(THT)
      EH1 = (1-(3*P2/(1-P2))**.5)*COSH(THT)*SIN(THT)
      EH = -2*(EH0+EH1)/DEN
      EQ = -(SINH(THT)*COS(THT)+COSH(THT)*SIN(THT))/DEN
      ET = -(SINH(THT)*COS(THT)-COSH(THT)*SIN(THT))/DEN
      SUBDEN = (.85-EB)/(1+BETA)
      EF = 1. + 2*EQ*SUBDEN
      EX = 1. +EH*SUBDEN
      EZ = .5+3.62*ET*SUBDEN
      PTOK = 24*E*Y1I/(DM**3*EL*(1-P2))
      PWIND = 2.6*E*(T/DM)**2.5/(EL/DM-.45*(T/DM)**.5)
      DENONE = (1+(.85*BETA/EB))/(1+BETA)
      DENUM = 2*T*Y/DM
      PY88 = 24*E*YBI/(DB**3*B*DENONE)
      SI92 = 1/(.5+1.81*EK*SUBDEN)
      SI92A = 1./(1+EH*SUBDEN)
      SIWENK = 1./(EF**2-EF/2+.25)**.5
      PLUNCH = 2*T*Y/D/(EX**2-EX*EZ+EZ**2)**.5
      PBRY = 2*E*T/DM/850. + 64*E*Y1I/(DM**3*EL)
      SIGOOR = PC*D*B*DENONE/(AB*2)
      SIGOOR = SIGOOR + E*((D-DB)/2.)*4*(PC/(PBRY-PC))/RONE**2
      SIGF = 1000*DM/(4*T)
```

```
      SIGT = SIGF*2*(1.+2*EQ*SUBDEN)
      SIGBT = SIGF*2*ET*SUBDEN
      SIGFOT = SIGF/SIGT
      SIGBOT = SIGBT/SIGT
      R = D/2.
      PI = 3.14159
      VT = PI*(EL*(R**2-(R-T)**2+B*((R-T)**2-(R-T-W)**2)
     1 +F*((R-T-W)**2-(R-T-W-T2)**2))
      WT = VT*.284*12/EL
      VOL = PI*(R/12.)**2*12
      SF92A = SI92A*DENUM*VOL/(WT*100000)
      SFWENK = SIWENK*DENUM*VOL/(WT*100000)
      Write statements for whatever solutions are required
  500 SR = SR+DSR
  700 PC = PC+100.0
      Format statements
      END
```

FUNCTION SUBROUTINES

```
      FUNCTION YENTRD(EL,T,B,W,T2,F)
  902 TOP = 2*F*T2*W + F*T2**2 + B*W**2 - EL*T**2
  903 BOT = 2*(F*T2 +B*W + EL*T)
  904 YENTRD = TOP/BOT
      RETURN
      END

      FUNCTION YMOM(EL,TLBLWLT2,F,Y)
  922 FIRST = (EL*T**3 + B*W**3 + F*T2**3)/12
  923 SECOND = EL*T*(T/2+Y)**2 + B*W*(Y-W/2)**2
  924 THIRD = F*T2*(T2/2+W-Y)**2
  925 YMOM = FIRST + SECOND + THIRD
      RETURN
      END
```

PRINTOUT OF PROGRAM 2 (Sample, continued on next page)

```
T IS  3.0INCHES AND T/DM IS 0.00629
LAMBDA,EL,FRAME FACTOR     0.380   12.883     4
SECOND MOMENTS:F,B,1   200.436   439.394   1481.279
 F AREA    WINDENBG    FORM88    FORM92     FORM92A    WENK
28.2750 -28193.383   590.321  1.857       1.488       1.665

LAMBDA,EL,FRAME FACTOR     0.400   14.275     4
SECOND MOMENTS:F,B,1   225.964   485.456   1680.225
 F AREA    WINDENBG    FORM88    FORM92     FORM92A    WENK
30.022  -42473.102   609.692  1.830       1.469       1.651

LAMBDA,EL,FRAME FACTOR     0.420   15.738     4
SECOND MOMENTS:F,B,1   253.112   533.708   1893.137
 F ARAE    WINDENBG    FORM88    FORM92     FORM92A    WENK
31.774  -90844.625   628.875  1.801       1.451       1.637

LAMBDA,EL,FRAME FACTOR     0.440   17.273     4
SECOND MOMENTS:F,B,1   281.881   584.134   2120.124
 F AREA    WINDENBG    FORM88    FORM92     FORM92A    WENK
33.531   467240.063  647.914  1.770       1.434       1.624

LAMBDA,EL,FRAME FACTOR     0.460   18.879     4
SECOND MOMENTS:F,B,1   312.279   636.729   2361.309
 F AREA    WINDENBG    FORM88    FORM92     FORM92A    WENK
35.293   62893.883   666.852  1.736       1.420       1.612

LAMBDA,EL,FRAME FACTOR     0.480   20.556     4
SECOND MOMENTS:F,B,1   344.308   691.478   2616.743
 F AREA    WINDENBG    FORM88    FORM92     FORM92A    WENK
37.058   33034.949   685.733  1.700       1.400       1.599

LAMBDA,EL,FRAME FACTOR     0.500   22.305     4
SECOND MOMENTS:F,B,1   377.968   748.369   2886.512
 F AREA    WINDENBG    FORM88    FORM92     FORM92A    WENK
38.828   22097.688   704.602  1.662       1.383       1.587

LAMBDA,EL,FRAME FACTOR     0.520   24.125     4
SECOND MOMENTS:F,B,1   413.264   807.395   3170.714
 F AREA    WINDENBG    FORM88    FORM92     FORM92A    WENK
40.600   16434.441   723.510  1.622       1.367       1.575

LAMBDA,EL,FRAME FACTOR     0.540   26.016     4
SECOND MOMENTS:F,B,1   450.198   868.543   3468.377
 F AREA    WINDENBG    FORM88    FORM92     FORM92A    WENK
42.376   12977.973   742.510  1.581       1.350       1.563

LAMBDA,EL,FRAME FACTOR     0.560   27.979     4
SECOND MOMENTS,F,B,1   488.769   931.803   3782.55
 F AREA    WINDENBG    FORM88    FORM92     FORM92A    WENK
44.154   10653.945   761.666  1.538       1.333       1.551
```

FLANGE,WEB:LENGTH,THICKNESS 9.80 1.96 7.84 1.15
LUNCHICK RATIOS:F/T,B/T .750 .0080

| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.629 | 2478.0 | 54958.6 | 26118.07 | 1.246 | 1.327 |

FLANGE,WEB:LENGTH,THICKNESS 10.10 2.02 8.08 1.19
LUNCHICK RATIOS,F/T,B/T .742 .0095

| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.610 | 2526.2 | 55407.4 | 25657.38 | 1.253 | 1.339 |

FLANGE,WEB:LENGTH,THICKNESS 10.39 2.08 8.31 1.22
LUNCHICK RATIOS:F/T,B/T .735 .0112

| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.592 | 2571.9 | 55852.6 | 25234.33 | 1.257 | 1.348 |

FLANGE,WEB:LENGTH,THICKNESS 10.68 2.13 8.54 1.26
LUNCHICK RATIOS:F/T,B/T .727 .0131

| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.574 | 2615.4 | 56292.1 | 24844.41 | 1.265 | 1.356 |

FLANGE,WEB:LENGTH,THICKNESS 10.95 2.19 8.76 1.29
LUNCHICK RATIOS:F/T,B/T .720 .0151

| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.556 | 2656.7 | 56722.0 | 24483.86 | 1.267 | 1.363 |

FLANGE,WEB:LENGTH,THICKNESS 11.22 2.25 8.98 1.32
LUNCHICK RATIOS:F/T,B/T .713 .0174

| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.538 | 2695.9 | 57140.1 | 24149.51 | 1.268 | 1.369 |

FLANGE,WEB:LENGTH,THICKNESS 11.49 2.30 9.19 1.35
LUNCHICK RATIOS:F/T,B/T .706 .0198

| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.519 | 2733.4 | 57544.6 | 23838.47 | 1.269 | 1.374 |

FLANGE,WEB:LENGTH,THICKNESS 11.75 2.35 9.40 1.38
LUNCHICK RATIOS:F/T,B/T .700 .0225

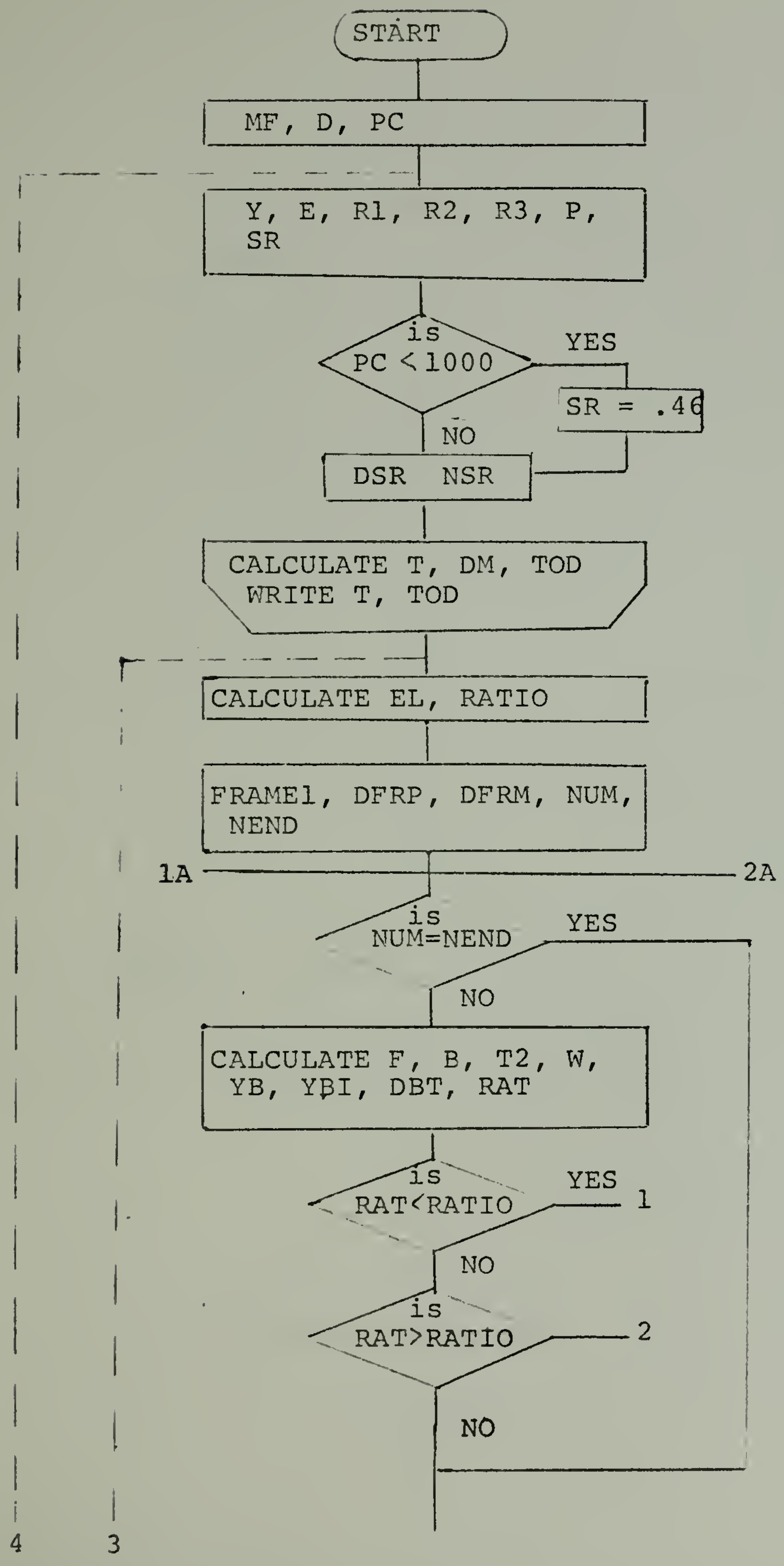
| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.501 | 2769.0 | 57932.6 | 23548.48 | 1.269 | 1.378 |

FLANGE,WEB:LENGTH,THICKNESS 12.00 2.40 9.60 1.41
LUNCHICK RATIOS:F/T,B/T .694 .0253

| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.482 | 2803.1 | 58301.6 | 23277.38 | 1.267 | 1.379 |

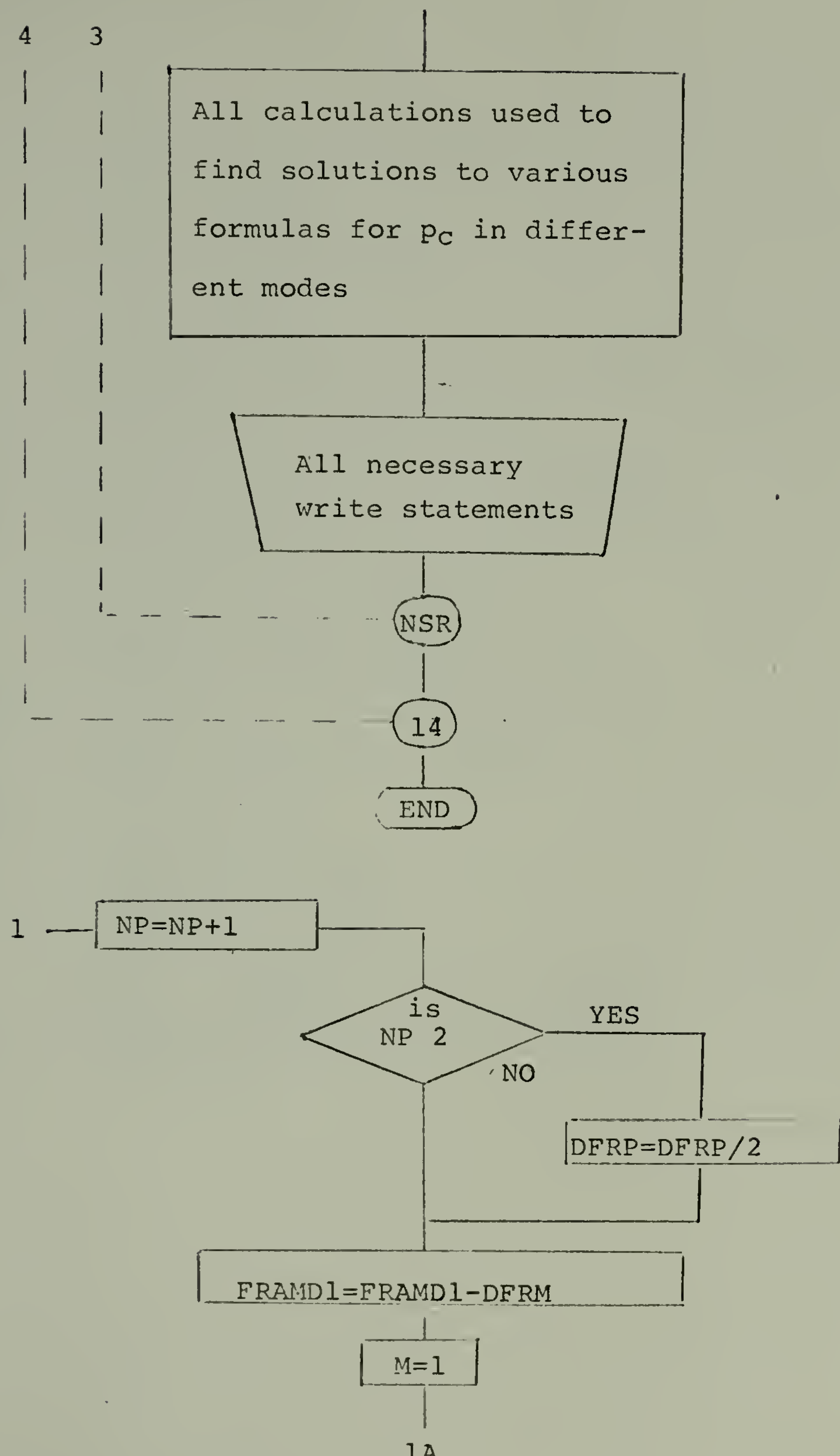
FLANGE,WEB:LENGTH,THICKNESS 12.25 2.45 9.80 1.44
LUNCHICK RATIOS:F/T,B/T .687 .0284

| LUNCHICK | BRYANT | OUT ROUND | WT/FT | EF92 | EFWENK |
|---|---|---|---|---|---|
| 1.462 | 2835.6 | 58649.1 | 23023.39 | 1.265 | 1.381 |




FLOW CHART

PROGRAM 2




Loop 2 to 2A is the same as 1 to 1A in its method

NOMENCLATURE OF PROGRAM 2

| | | |
|---|---|---|
| B | - | Width of frame in contact with shell |
| D | - | Outside diameter of shell |
| DSR | - | Incremental change in slenderness ratio |
| E | - | Young's Modulus |
| EL | - | Length of shell between frames |
| F | - | Frame overall width |
| FRAMD1 | - | Frame overall depth |
| MF | - | Frame Factor |
| NSR | - | Number of increments of slenderness ratio |
| P | - | Poisson's Ratio |
| PBRY | - | Collapse pressure due to Bryant's Formula |
| PC | - | Required collapse pressure |
| PLUNCH | - | Collapse pressure due to Lunchick |
| PTOK | - | Collapse pressure due to Tokugawa |
| PWIND | - | Collapse pressure due to Windenburg |
| PY88 | - | Collapse pressure due to #88 |
| R1 | - | Frame width/Frame depth |
| R2 | - | Web thickness/Frame Depth |
| R3 | - | Flange thickness/Web thickness |
| SF92A | - | Efficiency due to #92A |
| SFWENK | - | Efficiency due to Wenk's Formula |
| SIGFOT | - | $\sigma_F/\sigma_T$ for Lunchick Formula |
| SIGBOT | - | $\sigma_{BT}/\sigma_T$ for Lunchick Formula |
| SI92 | - | $\psi$ due to #92 |
| SI92A | - | $\psi$ due to #92A |
| SIWENK | - | $\psi$ due to Wenk |
| SR | - | Slenderness Ratio |
| T | - | Thickness of shell |
| T2 | - | Thickness of flange |
| W | - | Width of web |
| WT | - | Weight of vessel per foot length |
| VOL | - | Volume displaced per foot length |
| YENTRD | - | Subprogram to find neutral axis |
| YMOM | - | Subprogram to find I |

# APPENDIX G

## COMPUTED DATA

## and

## EXPERIMENTAL DATA FROM THE LITERATURE

TABLES OF MAXIMUM VALUES FROM FORMULA #92A AND MAXIMUM PRINCIPLE STRESS CRITERION

| | $100h/D_m$ | $\eta$ max/1.2 | $\lambda$ | $\psi$ |
|---|---|---|---|---|
| Frame Factor 4 | .251 | .9086 | .80 | 1.785 |
| | .313 | .9490 | .72 | 1.664 |
| | .376 | .9806 | .66 | 1.574 |
| | .439 | 1.0052 | .61 | 1.509 |
| | .503 | 1.0253 | .58 | 1.447 |
| | .566 | 1.0426 | .54 | 1.410 |
| | .629 | 1.0570 | .51 | 1.375 |
| | .692 | 1.0685 | .49 | 1.341 |
| | .756 | 1.0800 | .46 | 1.321 |
| | .819 | 1.0901 | .45 | 1.291 |
| | .883 | 1.0987 | .44 | 1.266 |
| | .946 | 1.1074 | .42 | 1.252 |
| | 1.010 | 1.1146 | .40 | 1.240 |
| Frame Factor 5 | .251 | .9317 | .80 | 1.716 |
| | .313 | .9706 | .72 | 1.602 |
| | .376 | .9994 | .66 | 1.519 |
| | .439 | 1.0224 | .61 | 1.458 |
| | .503 | 1.0411 | .57 | 1.409 |
| | .566 | 1.0570 | .54 | 1.367 |
| | .629 | 1.0699 | .51 | 1.334 |
| | .692 | 1.0814 | .49 | 1.304 |
| | .756 | 1.0916 | .47 | 1.278 |
| Frame Factor 6 | .251 | .9504 | .82 | 1.642 |
| | .313 | .9864 | .72 | 1.554 |
| | .376 | 1.0138 | .65 | 1.485 |
| | .439 | 1.0354 | .61 | 1.419 |
| | .503 | 1.0541 | .57 | 1.373 |
| | .566 | 1.0685 | .54 | 1.334 |
| | .629 | 1.0814 | .51 | 1.305 |
| | .692 | 1.0916 | .49 | 1.276 |
| | .756 | 1.1016 | .47 | 1.253 |
| | .819 | 1.1102 | .45 | 1.234 |
| | .883 | 1.1175 | .43 | 1.219 |
| | .946 | 1.1246 | .42 | 1.201 |
| | 1.010 | 1.1304 | .40 | 1.191 |

TABLES OF MAXIMUM VALUES FROM WENKS FORMULA
HENCKY-VON MISES YIELD CRITERION-MIDPLANE, MIDBAY

| | $100h/D_m$ | $\eta$ max/1.2 | $\lambda$ | $\psi$ |
|---|---|---|---|---|
| Frame Factor 4 | .376 | 1.1333 | .82 | 1.646 |
| | .439 | 1.164 | .76 | 1.611 |
| | .503 | 1.198 | .72 | 1.562 |
| | .566 | 1.214 | .67 | 1.533 |
| | .629 | 1.237 | .63 | 1.506 |
| | .692 | 1.248 | .60 | 1.481 |
| | .756 | 1.265 | .58 | 1.454 |
| | .819 | 1.270 | .55 | 1.438 |
| | .883 | 1.280 | .52 | 1.425 |
| | .946 | 1.292 | .50 | 1.409 |
| | 1.010 | 1.303 | .49 | 1.391 |
| Frame Factor 5 | .376 | 1.1592 | .84 | 1.620 |
| | .439 | 1.1880 | .76 | 1.571 |
| | .503 | 1.2139 | .70 | 1.538 |
| | .566 | 1.2341 | .67 | 1.497 |
| | .629 | 1.2500 | .63 | 1.472 |
| | .692 | 1.2644 | .60 | 1.448 |
| | .756 | 1.2759 | .57 | 1.428 |
| Frame Factor 6 | .376 | 1.1794 | .84 | 1.573 |
| | .439 | 1.2068 | .76 | 1.540 |
| | .503 | 1.2298 | .71 | 1.502 |
| | .566 | 1.2471 | .66 | 1.476 |
| | .629 | 1.2609 | .62 | 1.451 |
| | .692 | 1.2744 | .60 | 1.423 |
| | .756 | 1.2859 | .57 | 1.405 |
| | .819 | 1.2960 | .54 | 1.390 |
| | .883 | 1.3032 | .52 | 1.374 |
| | .946 | 1.3118 | .51 | 1.356 |
| | 1.010 | 1.3176 | .49 | 1.345 |

FRAME PARAMETERS AT MAXIMUM EFFICIENCY #92A

| | $100h/D_m$ | | $I_1/I_F$ | $100b/D$ |
|---|---|---|---|---|
| Frame Factor 4 | .251 | .80 | 4.758 | .2195 |
| | .313 | .72 | 5.335 | .2350 |
| | .376 | .66 | 5.860 | .2485 |
| | .439 | .61 | 6.340 | .2640 |
| | .503 | .58 | 6.817 | .2720 |
| | .566 | .54 | 7.238 | .2780 |
| | .629 | .51 | 7.655 | .2850 |
| | .692 | .49 | 8.072 | .2930 |
| | .756 | .46 | 8.464 | .2960 |
| | .819 | .45 | 8.871 | .3045 |
| | .883 | .44 | 9.267 | .3120 |
| | .946 | .42 | 9.662 | .3145 |
| | 1.010 | .40 | 10.065 | .3150 |
| Frame Factor 5 | .251 | .80 | 5.064 | .2065 |
| | .313 | .72 | 5.672 | .2215 |
| | .376 | .66 | 6.224 | .2340 |
| | .439 | .61 | 6.735 | .2420 |
| | .503 | .57 | 7.212 | .2525 |
| | .566 | .54 | 7.679 | .2610 |
| | .629 | .51 | 8.124 | .2675 |
| | .692 | .49 | 8.567 | .2745 |
| | .756 | .47 | 9.001 | .2810 |
| Frame Factor 6 | .251 | .82 | 5.364 | .1990 |
| | .313 | .72 | 5.958 | .2110 |
| | .376 | .66 | 6.533 | .2220 |
| | .439 | .61 | 7.062 | .2315 |
| | .503 | .57 | 7.566 | .2400 |
| | .566 | .54 | 8.056 | .2475 |
| | .629 | .51 | 8.527 | .2535 |
| | .692 | .49 | 8.995 | .2605 |
| | .756 | .47 | 9.451 | .2660 |
| | .819 | .45 | 9.919 | .2700 |
| | .883 | .43 | 10.391 | .2725 |
| | .946 | .42 | 10.853 | .2785 |
| | 1.010 | .40 | 11.354 | .2790 |

FRAME PARAMETERS AT MAXIMUM EFFICIENCY #WENK

| | $100h/D_m$ | | $I_1/I_F$ | 100b/D |
|---|---|---|---|---|
| Frame Factor 4 | .376 | .82 | 6.192 | .2800 |
| | .439 | .76 | 6.647 | .2925 |
| | .503 | .72 | 7.085 | .3055 |
| | .566 | .67 | 7.468 | .3140 |
| | .629 | .63 | 7.843 | .3210 |
| | .692 | .60 | 8.213 | .3290 |
| | .756 | .58 | 8.580 | .3375 |
| | .819 | .55 | 8.930 | .3420 |
| | .883 | .52 | 9.280 | .3440 |
| | .946 | .50 | 9.634 | .3495 |
| | 1.010 | .49 | 9.986 | .3555 |
| Frame Factor 5 | .376 | .84 | 6.568 | .2670 |
| | .439 | .76 | 7.008 | .2755 |
| | .503 | .70 | 7.433 | .2830 |
| | .566 | .67 | 7.878 | .2950 |
| | .629 | .63 | 8.264 | .3010 |
| | .692 | .60 | 8.657 | .3090 |
| | .756 | .57 | 9.041 | .3140 |
| Frame Factor 6 | .376 | .84 | 6.853 | .2540 |
| | .439 | .76 | 7.311 | .2620 |
| | .503 | .71 | 7.768 | .2715 |
| | .566 | .66 | 8.195 | .2780 |
| | .629 | .62 | 8.616 | .2835 |
| | .692 | .60 | 9.037 | .2930 |
| | .756 | .57 | 9.446 | .2980 |
| | .819 | .54 | 9.859 | .3010 |
| | .883 | .52 | 10.273 | .3060 |
| | .946 | .51 | 10.678 | .3130 |
| | 1.010 | .49 | 11.106 | .3160 |

EXPERIMENTAL DATA FROM REFERENCE 9

| Model Number | 100h/D | $\sigma_Y/1000$ | $\lambda$ | $\psi_{92A}$ | $\psi_{exp}$ |
|---|---|---|---|---|---|
| 1 | .88 | 46.8 | .41 | 1.43 | 1.68 |
| 2 | .784 | 47.5 | .42 | 1.42 | 1.74 |
| BR7m | .784 | 59.2 | .46 | 1.44 | 1.62 |
| BR7 | .599 | 44.8 | .59 | 1.11 | 1.3 |
| 5 | .814 | 47.1 | .61 | 1.01 | 1.24 |
| 6 | .474 | 49.6 | .67 | 1.09 | 1.24 |
| 7 | .581 | 47.0 | .70 | .997 | 1.11 |

$\psi_{exp}$ represents $\psi$ determined using actual collapse pressure and $\psi_{92A}$ was determined using collapse pressure calculated by formula #92A.

EXPERIMENTAL DATA FROM REFERENCE 19

General Model Parameters

$L_{eft}/D_m$ = .1405

E = 30,000,000 $lb/in^2$

Light Frames

| MODEL NUMBER | 100h/D | $\sigma_y/1000$ | $\lambda$ | $\psi_{exp}$ |
|---|---|---|---|---|
| 79 | .301 | 27 | .876 | .92 |
| 82 | .303 | 28 | .855 | .90 |
| 76 | .307 | 42 | 1.06 | .66 |
| 88 | .293 | 27 | .890 | .85 |
| 75 | .294 | 39 | 1.06 | .91 |
| 81 | .295 | 28 | .900 | .93 |
| 84 | .300 | 29 | .909 | .81 |
| 87 | .300 | 29 | .905 | .81 |
| 80 | .310 | 28 | .840 | .87 |
| 85 | .311 | 29 | .884 | .83 |
| 92 | .288 | 32 | 1.00 | .85 |
| 91 | .290 | 32 | .975 | .84 |
| 93 | .308 | 29 | .886 | .88 |
| 96 | .298 | 28 | .896 | .85 |
| 94 | .298 | 28 | .896 | .84 |
| 90 | .300 | 27 | .88 | .82 |

In the computation of $\lambda$ values, h/D was used instead of $h/D_m$ resulting in a small error which was not critical in the graphical methods used.

$\psi_{exp}$ represents the ratio of actual collapse pressure to the hoop stress collapse pressure.

ALUMINAUT

In the following section, $\lambda$, $\psi$, and $\eta$ have been found for the pressure hull in Aluminaut. The dimensions of that craft were taken from References (5) and (15) and are for the cylindrical hull alone.

| | | | | | |
|---|---|---|---|---|---|
| Operating Depth | = | 15,000 ft. | LOA | = | 33.3 ft. |
| Safety Factor | = | 1.4 | $L_e$ | = | 6.6 ft. |
| $\sigma_Y$ | = | 60,000 psi | D | = | 8.0 ft. |
| E | = | 29,000,000 psi | $D_m$ | = | 7.5 ft. |
| $\rho$ | = | .1 lb/in$^3$ | h | = | .5 ft. |

1. $\lambda = \sqrt[4]{\dfrac{(L/D_m)^2}{(h/D_m)^3}}\sqrt{\dfrac{\sigma_Y}{E}}$

$\lambda = .324$

2. $\psi = \dfrac{p_c}{2\frac{h}{D}\sigma_Y}$ $\qquad p_c = 1.4 \times 15{,}000 \times .446$

$p_c = 9400$ psi

$\psi = 1.25$

3. $\eta = \dfrac{p_c V}{W\,10^4}$

$\eta = 2.65$ in.

From the above computations, it is apparent that the designers of Aluminaut produced a vessel whose parameters put it in the low lambda design region. However, it has been noted (15) that a more efficient structure is possible.

Thesis 134735
H5264 Hickey
An investigation of low lambda yield failure design of stiffened cylindrical pressure vessels under hydrostatic pressure.